Ref. No. EX/ME/MATH/T/121/2017(OLD)

B.E. MEC 1ST YR, 2ND SEM EXAM, 2017(OLD)
MATHEMATICS- III M (OLD)

TIME: FULL MARKS: 50

Instructions: 1. Use separate answer scripts for each group
2. Answer any five questions

PART : I

1. a) Show that

i) $L\{t^n\} = \frac{n!}{s^{n+1}}$.

ii) $L(e^{at}) = \frac{1}{s-a}, if\ s > a.$

b) Find the Laplace transform of $\int_0^1 (\frac{1-e^{-2x}}{x})dx.$ $(3 \times 2) + 4$

2. Find the Fourier series to represent $f(x) = x^2 - 2$ in interval $-2 < x < 2$. 10

3. Find Fourier series for $f(x) = x$ in $0 < x < \pi$. 10

4. a) Find the Fourier cosine transform of e^{-x^2} and hence evaluate Fourier sine transform of xe^{-x^2}.

b) Solve $\partial u/\partial t = 2\,\partial u^2/\partial x^2$, if $u(0,t) = 0, u(x,0) = e^{-x}\ (x > 0)$, $u(x,t)$ is bounded where $x > 0, t > 0$. 5+5

5. State Dirichlet's conditions. Find the Fourier series expansion for f(x), if

$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi. \end{cases}$$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$. 10

6. a) Find the Laplace transform of the function

$$f(t) = \begin{cases} \sin wt, & 0 < t < \frac{\pi}{w} \\ 0, & \frac{\pi}{w} < t < \frac{2\pi}{w}. \end{cases}$$

b) Find the Laplace transforms of $t^2 e^{-3t} \sin 2t$.

c) Apply Convolution theorem to evaluate $L^{-1}\{\frac{s}{(s^2+1)(s^2+4)(s^2+9)}\}$. 3+3+4

7. a) Find the inverse Laplace transforms of the following:

i) $\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}$, ii) $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$.

b) Solve by using Laplace transformation: $(D^2 + 9)y = \cos(2t)\ if\ y(0) = 1, y\left(\frac{\pi}{2}\right) = -1.$ (3×2)+4

(50 marks for each Part)

PART - II

Answer ***question no.7*** and ***any three*** from the rest.

7. Write down the axiomatic definition of probability and show that classical definition is a particular case of aximatic definition. 2

8. a) Explain the following terms with examples :

   i) random experiment ii) sample space iii) event 6

   b) Prove that for n events $A_1, A_2, ..., A_n$,

$$\sum_{i=1}^{n} P(A_i) - \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i<j}}^{n} P(A_i \cap A_j) \leq P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$ 10

9. a) A five-figure number is formed by the digits 0, 1, 2, 3, 4 (without repetition.) Find the probability that the number formed is divisible by 4. 5

   b) State and prove Baye's Theorem. 5

   c) There are three .... having the following compositions of black and white balls :