BACHELOR OF ENGINEERING IN MECHANICAL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester)

MATHEMATICS - II

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Answer ***any five*** questions. 5×10=50

1. a) Find the matrix A if $\operatorname{adj} A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and detA=2. 5

   b) Show that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3.$$ 5

2. a) Find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 4 \\ 1 & 0 & 1 \\ 0 & -1 & 4 \end{pmatrix}$. Hence solve the following system of equations : 5

$$\begin{aligned} 2x - 3y + 4z &= -4 \\ x \qquad + z &= 0 \\ -y + 4z &= 2 \end{aligned}$$

[Turn over

b) Find the rank of the matrix 5

$$\begin{pmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{pmatrix}$$

3. a) i) If A and B are two symmetric matrices then show that (AB+BA) is a symmetric matrix and (AB–BA) is a skew-symmetric matrix.

ii) Express the matrix $A = \begin{pmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{pmatrix}$ as the sum of a symmetric matrix and a skew-symmetric matrix.

2+3=5

b) When the matrix equation AX=B has a non zero solution? If the system of equations ax+by+cz=0, bx+cy+az=0 and cx+ay+bz=0 have a non-zero solution then show that either a+b+c=0 or a=b=c. 1+4=5

4. a) Find the eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$ 5

10. a) If $\vec{u}$ be any given continuously differentiable vector point function, then prove that $\nabla \times (\nabla \times \vec{u}) = \nabla(\nabla \cdot \vec{u}) - \nabla^2 \vec{u}$.

b) What are the values of a, b and c so that

$$\vec{F} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$

is irrotational ? 5+5

11. Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$, where $\vec{F} = z\hat{i} + x\hat{j} - 3y^2 z\,\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z=0 and z=5. 10

12. State and prove Green's theorem in the plane. 10

13. Verify Gauss' divergence theorem for the vector function $\vec{F} = 2xz\,\hat{i} + y\,\hat{j} + yz\,\hat{k}$ taken over the surface of the cube bounded by x=0, y=0, z=0 and x=1, y=1, z=1. 10

14. Verify Stoke's theorem for the vector

$\vec{F} = (2x - y)\,\hat{i} - yz^2\,\hat{j} - y^2 z\,\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

10

7. a) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point (2, –4, 6). 5

b) Obtain the equation of the circle lying on the sphere $x^2+y^2+z^2-2x+2y-4z+3=0$ and having its centre at the point (2, 2, –3). 5

PART - II

Answer ***any five (5)*** questions.

8. a) Find the vector area and scalar area of the triangle, the position vectors of whose vertices are $\left(\hat{i}+\hat{j}+2\hat{k}\right)$, $\left(2\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\left(3\hat{i}-\hat{j}-\hat{k}\right)$.

b) Find a vector $\vec{\delta}$, which is perpendicular to both the vectors $\vec{\alpha}=4\hat{i}+5\hat{j}-\hat{k}$ and $\vec{\beta}=\hat{i}-4\hat{j}+5\hat{k}$ and satisfies $\vec{\delta}\cdot\vec{\gamma}=2$, where $\vec{\gamma}=3\hat{i}+\hat{j}-\hat{k}$. 5+5

9. a) Find the expressions for torsion and curvature at a point on the circular helix $\vec{r}=a\left(\cos\theta, \sin\theta, \theta\cot\beta\right)$.

b) Determine the directianal derivative of

$\phi\left(x, yz\right)=xy^2z+4x^2z$ at (–1, 1, 2) in the direction $\left(2\hat{i}+\hat{j}-2\hat{k}\right)$. 5+5

b) Show that the matrix $A=\begin{pmatrix}1 & 2 & 1\\ 1 & -1 & 1\\ 2 & 3 & -1\end{pmatrix}$ satisfies Cayley-Hamiltor theorem. Hence find the inverse of A if it exists. 5

5. a) Find the equation of the plane which is perpendicular to the plane x+2y–z+1=0 and which contains the line of intersection of the planes x+2y+3z–4=0 and 2x+y+z+2=0. 5

b) A variable plane at a constant distance p from the origin 0 meets the axes at A, B, C. Show that the locus of the centroid of the tetrahedron 0ABC is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}.$$ 5

6. a) Find the equations of projection of the line $3x-y+2z-1=0=x+2y-z-2$ on the plane $3x+2y+z=0$ in the symmetrical form. 5

b) Find the shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. Find also the equations and the points of intersection in which it meets the lines. 5