

(2<sup>nd</sup> Semester)

## FORMAL LANGUAGE AND AUTOMATA THEORY

Time: Three hours

Full Marks: 100

Attempt any **FIVE** questions

1) Answer the following questions.

- (a) Define a Deterministic Finite Automata (DFA).  
 (b) Design a DFA,  $D$ , to recognize the Language  $L = \{W \in (a,b)^* \mid w \text{ starts and ends with the } aa \text{ or } bb\}$   
 (c) From the DFA of the question 1 (a), define its regular grammar.  
 (d) State the pumping lemma for regular languages.

3+8+5+4

2) Answer the following questions.

(a) Consider the following grammar.

$$S \rightarrow aaB \mid Abb$$

$$A \rightarrow a \mid aA$$

$$B \rightarrow b \mid bB$$

i) Which language does this grammar generate? Justify.

ii) Prove that the grammar is ambiguous

iii) Define an unambiguous grammar equivalent to the one given in this Question.

(b) Define Chomsky Normal Form (CNF) of a Context Free Grammar. What are the steps to convert a general Context Free Grammar to a CNF?

(4+6+6)+(2+2)

3) Answer the following questions.

(a) State the Pumping Lemma for Context Free Grammars.

(b) One of the following is a Context Free Language and the other is not. For the one, which is Context Free, define the Grammar. For the one which is not a Context Free Language, prove it by Pumping Lemma.

$$i) L = \{a^l b^m c^n \mid l, m, n > 0, l + m \geq n\}.$$

$$ii) L = \{a^l b^m c^n \mid l, m, n \geq 0, l \geq n \text{ and } m \geq n\}$$

4+(8+8)

4) Answer the following questions:

(a) When is a string accepted by a Push Down Automata (PDA)?

(b) Construct a PDA that accepts all binary strings that contain an equal number of a's and b's

(c) Construct an NPDA for the following Context Free Grammar.

$S \rightarrow aABB \mid aAA$

$A \rightarrow aBB \mid a$

$B \rightarrow bBB \mid A$

(d) Show that Context Free Languages are not closed under intersection.

3+8+7+2

5) Answer the following questions.

(a) Define a standard Turing Machine.

(b) Define a Turing Machine that accepts a Language  $L = \{w\#w \mid w \in \{0,1\}^*\}$ .  
The TM may erase the content of the tape while processing.

(c) Construct a Turing Machine that accepts the language  $L = \{w \in \{a,b,c\}^* \mid \#a \leq \#b \leq \#c\}$ .

4+8+8

6) Answer the following questions.

(a) Consider the following Context Sensitive Grammar.

$S \rightarrow AS \mid aT$

$Aa \rightarrow aaaA$

$AT \rightarrow T$

$T \rightarrow \epsilon$

i) Show a derivation of  $a^9$ .

ii) Which language does the grammar generate? Explain

(b) How can you encode Turing Machine in a string of 1's and delimiters? Explain by an example.

(c) How problem reduction helps in proving that a problem is undecidable?

(3+2+5)+5+5

7) Answer the following questions:

(a) Give one example of a language which is Context Free but not Deterministically Context Free. Can you justify that you have given a correct example?

(b) What is a recursively enumerable language?

(c) If  $R_1$  and its complement are Recursively Enumerable languages, the language  $R_1$  is recursive – Justify or contradict.

(d) What is meant by an undecidable problem? Give one example of an undecidable problem.

(2+6)+3+5+(2+2)