

2nd year, 2nd semester)

Time: Three hours

Full Marks: 100

NUMERICAL METHODS & OPTIMIZATION TECHNIQUES

PART-I

Group-A

10*1=10

1. (i) In Simpson's 1/3rd rule of finding $\int_a^b f(x)dx$ $f(x)$ is approximated by

- (a) line segment (b) parabola
(c) Circular sector (d) Part of ellipse

(ii). Which of the following relations is true ?

- (a) $E = 1 + \Delta$
(b) $E = 1 - \Delta$
(c) $E = 1/\Delta$
(d) none of these

(iii). Newton-Raphson method fails when

- (a) $f'(x) = 1$
(b) $f'(x) = 0$
(c) $f'(x) = -1$
(d) $f''(x) = 0$

(iv). Condition of convergence for Euler's method

- (a) $|1 + hf'(x_i, y_i)| < 1$
(b) $|1 + hf'(x_i, y_i)| \leq 1$
(c) $|1 + hf'(x_i, y_i)| > 1$
(d) $|1 + hf'(x_i, y_i)| \geq 1$

(v). Divided difference interpolation formula can be used for

- (a) The tabular values with independent variable unequally spaced
(b) Inverse interpolation
(c) both (a) and (b)
(d) None of these

(vi). Gauss Seidel method for solution of a system of linear simultaneous equations converges if

- (a) $|a_{ii}| \geq |a_{ij}|$, where $j \neq i$

(b) $|a_{ii}| > |a_{ij}|$, where $j \neq i$

(c) $|a_{ii}|/|a_{nn}|=1$

(d) None of these

(vii). Ragula_falsi method has a convergence rate of the order of

(a) 2 (b) 1.62 (c) 1 (d) None of these

(viii). Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If iterations are started from -1, then iterations will be

(a) converge to -1

(b) converge to $\sqrt{2}$

(c) converge to $-\sqrt{2}$

(d) no coverage

(ix). The degree of precision of Simpson's one third rule is

(a) 1 (b) 2 (c) 3 (d) 5

(x). Lagrange's interpolation formula deals with

(a) Equispaced arguments only

(b) Unequispaced argument only

(c) both (a) and (b)

(d) None of these

Group-B Attempt any three (3):

3*5=15

2. Find the positive real root of $x^3 - x^2 - 1 = 0$, using the bisection method of 4 iterations. (5)

3. Estimate the value of the integral by Simpson's 1/3rd rule taking 4 strips

$$\int_1^3 \frac{1}{x} dx \quad (5)$$

4. Solve by using Euler's Method the following differential equation for $x=1$ by taking $h=0.2$

$$dy/dx=xy, y=1, \text{ when } x=0 \quad (5)$$

5. From the following table find the values of $f(12)$ by Newton's divided difference interpolation formula:

x:	11	13	14	18	19	21
y:	1342	2210	2758	5850	6878	9282

(5)

Attempt any five (5) questions:

6.(a) Solve the given system of equations:

$$10x+2y+z = 9$$

$$2x+20y-2z = -44$$

$$-2x+3y+10z = 22$$

(8)

by Gauss Seidel method.

(b) Derive Newton's Back ward difference interpolation formula.

(5)

(c) Define Δ , E

(2)

7. (a) Solve the following system of equations using LU factorization method.

$$3x-y+2z = 12$$

$$x+2y+3z = 11$$

$$2x-2y-z = 2$$

(8)

(b) Find the inverse of the following matrix.

$$A = \begin{vmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{vmatrix}$$

(7)

8. (a) Find Lagrange's interpolation polynomial passing through the set of points:

x:	0	1	2
y:	4	3	6

use it to find y at $x=1.5$ dy/dx at $x=0.5$ and evaluate $\int_0^3 y dx$ (8)

(b) Find the value of $\log 2^{1/3}$ from $\int_0^1 f(x) dx$, where $f(x)=x^2/(1+x^3)$ using Simpson's 1/3rd rule

with $h=0.25$

(7)

9. (a) Evaluate $\int_0^1 f(x)dx, f(x) = xe^x$ where the interval (0,-1) by using Trapezoidal rule taking $n=6$

(5)

(b) Find the Newton-Raphson iteration formula to find the pth root of positive number

N and hence find the cube root of 17.

(5)

(c) Solve the following equation using bisection method:

$$3x + \sin x - e^x = 0$$

take $x_0=1$ and $x_1=0$ result is required to be corrected upto 2 decimal places. (5)

10. (a) Find the polynomial $f(x)$ and hence calculate $f(5.5)$ for the given data

x:	0	2	3	5	7	
y:	1	47	97	251	477	

(6)

(b) Find a root of the equation $x \sin x + \cos x = 0$ using Newton-Raphson method correct

upto 5 places of decimal.

(6)

(c) Define advantage and disadvantage of Newton-Raphson method.

(3)

11. (a) Using Newton's forward formula compute y_{12} given that $y_{10} = 600, y_{20} = 512, y_{30} = 439,$

$$y_{40} = 346, y_{50} = 243.$$

(7)

(b) Solve the following initial value problem using Euler's method $dy/dx = x^2 + y$ with

$y(0) = 1$ compute the first 5 steps of the solution with $h=0.1$

(8)