Bachelor of Information Technology
2nd Year 2nd Semester Examination 2017

Time 3 Hours **Subject – Mathematics IV (Modules 8 & 12)** **Full Marks 100**

{Answer any ten questions}

1. i) Define independence of two events.
   ii) State and Prove Bayes' theorem on conditional probability.
   iii) Write down the expression of probability density function for Normal distribution. [2+6+2]

2. i) Can a function of the form
$$F(x) = \begin{cases} c\left(\frac{2}{3}\right)^x, & x = 1,2,3 \ldots. \\ 0, & elsewhere \end{cases}$$
   be a probability mass function? .
   ii) Determine the value of the constant C such that f(x) defined by
$$f(x) = \begin{cases} Cx(1-x), & 0 < x < 1, \\ 0, & elsewhere \end{cases}$$
   is a probability density function. Find the corresponding distribution function. [3+7]

3. i) The random variable X is uniformly distributed in (0, 1). Find the density function of Y= - 2 $\log_e$ X.
   ii) Evaluate the variance of Poisson distribution. [4+6]

4. i) Let X be a random variable with probability distribution as follows :

x :	0	1	2	3
$f(x)$:	$\frac{1}{3}$	$\frac{1}{2}$	0	$\frac{1}{6}$

   Find E[$(X - 1)^2$].
   ii) Suppose that, for a discrete random variable X, E(X)=2 and E[X(X - 4)]=5. Find the variance and standard deviation of $(-4X + 12)$. [5+5]

5. i) Define covariance between two random variables X and Y.
   ii) Prove that $-1 \leq \rho(X,Y) \leq 1$, where $\rho(X,Y)$ is the correlation coefficient between two random variables X and Y. [3+7]

6. i) The random variables X, Y are connected by the linear relation 2X+3Y+4=0. Show that $\rho(X,Y) = -1$.

   ii) If X be any continuous random variable having finite variance σ^2 (and hence having finite mean m), then for any $\varepsilon > 0$, prove that
$$P(|X - m| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$
   [4+6]

7. Define composite mapping. Let $f: \boldsymbol{R} \to \boldsymbol{R}$ defined by $f(x) = e^x, \forall x \in \boldsymbol{R}$ and let $g: \boldsymbol{R} \to \boldsymbol{R}$ defined by $g(x) = \sin x, \forall x \in \boldsymbol{R}$. Verify : $g_\circ f \neq f_\circ g$ and mention the range of f. [10]

8. State the principle of mathematical induction and prove that $n^3 + 5n$ is divisible by 6 for all $n \in \mathbf{N}$, by the principle of mathematical induction. [10]

9. Are the two functions f and g equal ? Give reasons.
Where $f : \boldsymbol{D} \to \boldsymbol{R}$ defined by $f(x) = \sin x - \cos x$, $x \in \boldsymbol{D}$ and $g : \boldsymbol{D} \to \boldsymbol{R}$ defined by $g(x) = \sqrt{(1 - \sin 2x)}$, $x \in \boldsymbol{D}$ and $\boldsymbol{D} = \{ x \in \boldsymbol{R} : 0 \leq x \leq \pi/2 \}$. [10]

10. What is injective mapping and prove that the function $f : \boldsymbol{R} \to \boldsymbol{R}$ defined by $f(x) = \frac{|x|}{|x| + 1}$, $x \in \boldsymbol{R}$ is neither injective nor surjective. [10]

11. What is enumerable set, give an example and show that a relation on a set which is transitive but neither reflexive nor symmetric. [10]

12. Define an ordered set and prove that, in an ordered set (X , <) if a subset S has a supremum x^*, then x^* is unique. [10]