EX/ IT/MATH/T/114/2017(S)

BACHOLOR OF INFORMATION TECHNOLOGY ENGG.
SUPPLEMENTARY EXAMINATION - 2017
(1^{ST} YR. 1^{ST}SEM.)
MATHEMATICS-I-(MODULE I & II)

Time: Three hours Full Marks: 100

Answer any **Ten** questions 10×10

1. Prove that the sequence $\{u_n\}$ defined by $u_1 = \sqrt{2}$ and $u_{n+1} = \sqrt{2u_n}$ for all $n \geq 1$ converges to 2. 10
2. To discuss the behaviour of the sequence $\{x_n\}$, where $x_n = \left(1 + \frac{1}{n}\right)^n$,(i)the sequence is monotone increasing (ii)the sequence $\{x_n\}$ is bounded. 10
3. (a)Test the convergence of the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots \ldots \ldots$
(b)Test the convergence of the series, $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \frac{7}{4.5.6} + \cdots \ldots \ldots \infty$ 10
4. Prove that the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots \ldots \ldots + \frac{1}{n^p} + \cdots \ldots \ldots$ converges if $p > 1$, diverges if $p \leq 1$. 10
5. If $y = \sin(msin^{-1} x)$, then show that (i) $(1 - x^2)y_2 - xy_1 + m^2 y = 0$; (ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$, 10
6. If $y = x^{n-1} \log x$, then show that $y_n = \frac{(n-1)!}{x}$ 10
7. (a)If $f(x) = (x - a)^m (x - b)^n$ where m and n are positive integers, show that c in Rolle's theorem divides the segment $a \leq x \leq b$ in the ratio $m : n$
(b)Use MVT, show that $\frac{\tan x}{x} > \frac{x}{\sin x}$ when $0 < x < \frac{\pi}{2}$. 10
8. If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(\theta h), 0 < \theta < 1$find θ when $h = 8, f(x) = \frac{1}{1+x}$ 10
9. Find the value of p and q in order that $\lim_{x \to 0} \frac{x(1+p\cos x) - q \sin x}{x^3} = 1$ 10
10. (If ρ_1 and ρ_2 be the radii of curvature at the ends of any focal chord of the parabola $y^2 = 4ax$, then find the value of $\rho_1^{-\frac{2}{3}} + \rho_2^{-\frac{2}{3}}$. 10
11. Determine the asymptotes of $x^3 + x^2 y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0$. 10
12. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then find the value of (i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$
(ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u$ and (iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ 10
13. If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, find the value of (i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ and (ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$ 10