EX/ IT/MATH/T/122/2017

BACHOLOR OF INFORMATION TECHNOLOGY ENGG.
EXAMINATION - 2017
(1ST YR. 2ND SEM.)
MATHEMATICS-II

Time: Three hours | Full Marks: 100

Answer any **Ten** questions | 10×10

1. Find the Fourier series of the function $f(x)$ defined by $f(x) = x \sin x$, $-\pi \le x \le \pi$. Hence deduce the value $\frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots \cdots \cdots \cdots \cdots$ 10
2. (a)Find the value of $L\left\{\frac{\cos at - \cos bt}{t} ; p\right\}$,using the property$L\left\{\frac{f(t)}{t} ; p\right\}$

   (b)Using Laplace Transform, find the value of $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}\, dt$ 5+5
3. (a)If $L^{-1}\left\{\frac{e^{-\frac{1}{p}}}{p^{\frac{1}{2}}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$, find the value of $L^{-1}\left\{\frac{e^{-\frac{a}{p}}}{p^{\frac{1}{2}}}\right\}$

   (b) Show that the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self –orthogonal, where λ is a variable parameter and a, b are constants. 5+5
4. Find the Fourier transform of $f(x)$, where $f(x) = 1 - x^2$, $|x| < 1$
   0 , $|x| > 1$

   and deduce the value of $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\frac{x}{2}\, dx$. 10
5. Represent $f(x)$,where $f(x) = \cos kx$, on $-\pi \le x \le \pi$ (k not being an integer) in Fourier series. Hence deduce that (i) $\pi \cot k\pi = \frac{1}{k} + 2k \sum_{n=1}^{\infty} \frac{1}{k^2 - n^2}$

   (ii) $\frac{\pi}{\sin k\pi} = \sum_{n=0}^{\infty} (-1)^n \left\{\frac{1}{n+k} + \frac{1}{n+1-k}\right\}$. 10
6. Find Fourier transform of$f(x)$,where$f(x) = \frac{\sin ax}{x}$ $a > 0$. 10
7. Using Perseval's relation for Fourier sine transform of the given function $f(x) = e^{-ax}$and$g(x) = e^{-bx}$, $a, b > 0$. Evaluate the integral $\int_0^\infty \frac{x^2}{(a^2+x^2)(b^2+x^2)}\, dx$. 10
8. (a)State and prove Existence theorem of Laplace Transform.

   (b)find the value of $L^{-1}\left\{\log\frac{p+b}{p+a}\right\}$, $a > 0, b > 0$ 6+4
9. Find the value of $L^{-1}\left\{\frac{2p+1}{(p+1)^2(P^2+1)(p^2+P+1)}\right\}$. 10
10. (a)Solve $x^3\, dy - \left(y^3 + y^2\sqrt{y^2 - x^2}\right)dx = 0$.

    (b)Solve $\frac{dy}{dx} + y\cos x = y^n \sin 2x$. 5+5
11. Solve by Laplace method, $y''(t) - 3y'(t) + 2y(t) = 4e^{2t}$, given that $y(0) = -3, y'(0) = 5$. 10
12. (i) Solve,$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ 5

    (ii) Solve $(D^3 + 1)y = e^{2x}\sin x + e^{\frac{x}{2}} \sin\frac{\sqrt{3}}{2}x$ 5
13. Solve the equation $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = x^2$ in series. 10