EX/ FTBE/MATH/T/224/2017

BACHOLOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. EXAM. - 2017

(2^{ND} YR. 2^{ND} SEM.)

MATHEMATICS-III

Time: Three hours | Full Marks: 100

GROUP-A

Answer any **five** questions $5 \times 2 = 10$

1. (a)Verify that the set of vectors $\vec{a} = (2, -1, 2)$, $\vec{b} = (1, 3, -2)$, $\vec{c} = (2, 4, 1)$ *and* $\vec{d} = (5, 3, -3)$ are linearly dependent or independent.

   (b)Determine the value of γ *and* μ for which the vectors $-3\vec{i} + 4\vec{j} + \gamma\vec{k}$ *and* $\mu\vec{i} + 8\vec{j} + 6\vec{k}$ are collinear.

   (c)What is the Fundamental theorem of integral calculus?

   (d)If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$, then show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar.

   (e) Calculate the absolute, relative and percentage errors by approximating $\frac{4}{3}$ by 1.333

   (f)Define rank of matrix.

   (g) Prove that, $\Delta log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$

GROUP-B

Answer any **Nine** questions $9 \times 10 = 90$

2. (a)Solve, $k\vec{r} + \vec{r} \times \vec{a} = \vec{b}$, where k is a non-zero scalar and $\vec{a}, \vec{b}$ are two given vectors.

   (b)Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$, under what condition $(\vec{b} \times \vec{c}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$? 6+4

3. (a)Expand by Laplace's method to evaluate $\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix}$

   (b)If $A = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{pmatrix}$ find the value of $A^2 - 10A + 16I_3$. Hence obtain A^{-1}. 6+4

4. (a)Prove that the volume of the solid obtained by revolving the lemniscates $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{1}{2}\pi a^3 \left\{\frac{1}{\sqrt{2}}\log(\sqrt{2}+1) - \frac{1}{3}\right\}$.

   (b)A particle, acted on by constant forces $4\vec{i} + \vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$, is displaced from the point $\vec{i} + 2\vec{j} + 3\vec{k}$ to the point $5\vec{i} + 4\vec{j} + \vec{k}$. Find the work done by the force on the particle. 6+4

[Turn over

5. (a)The smaller segment of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, cut off by the chord $\frac{x}{a}+\frac{y}{b}=1$ revolves completely about this chord, find the volume of the solid spindle thus generated.
   (b)Solve by Cramer's rule $x+y+z=1, ax+by+cz=1, a^2x+b^2y+c^2z=1, a\neq b\neq c$ 6+4

6. (a)Given two vectors $\vec{\alpha}=\vec{\imath}+2\vec{\jmath}-\vec{k},\ \vec{\beta}=2\vec{\imath}-\vec{\jmath}+\vec{k}$; find the vector $\vec{\gamma}$ and the scalar λ which satisfy $\vec{\alpha}\times\vec{\gamma}=\vec{\beta}+\lambda\vec{\alpha}$ and $\vec{\alpha}.\vec{\gamma}=2$.
   (b) Show that $\iint_R \sqrt{4a^2-x^2-y^2}\,dxdy=\frac{4}{9}(3\pi-4)a^3$ 5+5

7. (a)Show that the matrix $\begin{pmatrix}2&0&1\\3&3&0\\6&2&3\end{pmatrix}$ is non-singular and express it as a product of elementary matrices.
   (b)show that $[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$ 7 + 3

8. Obtain the fully reduced normal form of the matrix $\begin{pmatrix}0&0&1&2&1\\1&3&1&0&3\\2&6&4&2&8\\3&9&4&2&10\\4&12&4&5&11\\5&15&5&0&16\end{pmatrix}$. 10

9. (a)Solve the following system of equations

$$x+y+z=6$$
$$3x+(3+\epsilon)y+4z=20$$
$$2x+y+3z=13$$

   using the Gauss elimination method, where ϵ is small such that $1\pm\epsilon^2\cong 1$.
   (b)Prove that, $\Delta.\nabla=\Delta-\nabla=\nabla.\Delta$ 8+2

10. (a)Find the missing term in the following table:

X :	0	1	2	3	4	5
F(x)	0	-	8	15	-	35

    (b)What is the advantage's of Lagrange's formula for interpolation? 7+3

11. Compute by Simpsons one third rule $\int_{1.2}^{1.6}\left(x+\frac{1}{x}\right)dx$ correct to two significant figures, taking four intervals and compare the result with the actual value of the integral. 10

12. The following values of the function $f(x)$for value of x are given: $f(1)=4, f(2)=5, f(8)=4, f(9)=3$ *and* $f(10)=2$. Find the values of $f(6)$and also the value of x for which$f(x)$ is maximum or minimum. 10

13. (i) $\int_0^{\pi}\frac{x\,dx}{(a^2\cos^2x+b^2\sin^2x)^2}$ 6

    (ii) $\int\frac{dx}{(x^3+1)(x+2)}$ 4