25

EX/ FTBE/MATH/T/114/2017(S)

BACHOLOR OF ENGINEERING IN FOOD TECHNOLOGY AND BIOCHEMICAL ENGG. SUPPLE EXAMINATION - 2017

(1^{ST} YR. 1^{ST} SEM.)

MATHEMATICS-I

Time: Three hours | Full Marks: 100

Answer any **Ten** questions | 10×10

1. (i)If by a transformation of one rectangular axes to another with the same origin, the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.
(ii)Find the angle of rotation of the co-ordinate axes about the origin which will transform the equation $x^2 - y^2 = 4$ to $x'y' = 2$ 5+5

2. Prove that the transformation of rectangular axes which converts $\frac{x^2}{p} + \frac{y^2}{q}$ into $ax^2 + 2hxy + by^2$ will convert $\frac{x^2}{p-\gamma} + \frac{y^2}{q-\gamma}$ into $\frac{ax^2+2hxy+by^2-\gamma(ab-h^2)(x^2+y^2)}{1-(a+b)\gamma+(ab-h^2)\gamma^2}$. 10

3. In any conic, prove that

(i)the sum of the reciprocals of the segments of any focal chord is constant and

(ii) the sum of the reciprocals of two perpendicular focal chord is constant. 10

4. Find the polar equation of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$, if the pole is at right hand focus and the positive direction of the x-axis is the positive direction of the polar axis. 10

5. If l and l' are the lengths of the segments of any focal chord of the parabola $y^2 = 4ax$, prove that $\frac{1}{l} + \frac{1}{l'} = \frac{1}{a}$. 10

6. Find the equation of the common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$ and show that the two curves cut one another origin at an angle $\tan^{-1} \frac{3a^{\frac{1}{3}}b^{\frac{1}{3}}}{2\left[a^{\frac{2}{3}}+b^{\frac{2}{3}}\right]}$. 10

7. Show that the eccentricity of the ellipse in which the normal at one end of a latus rectum passes through one end of the minor axis is given by the equation $e^4 + e^2 - 1 = 0$ 10

8. Prove that the acute angle between the lines whose d. cs. are given by $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$ is $\frac{\pi}{3}$. 10

9. A variable plane passes through a fixed point (α, β, γ) and cuts the co-ordinate axes OX, OY and OZ in A, B and C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the coordinate planes is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$. 10

10. A variable plane makes intercepts on the coordinate axes, the sum of whose squares is constant and equal to k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is $\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)(x^2 + y^2 + z^2)^2 = k^2$. 10

11. Find the shortest distance between the lines $3x - 2y - 3z + 4 = 0 = x + 2y + z - 6$, $x + y - z = 0 = x - 2y + z$ and obtain the equations of the line along which the above shortest distance measured. 10

12. A plane passes through a fixed point (α, β, γ) and cuts the co-ordinate axes in A, B, C. Prove that the locus of the centre of the sphere $OABC$ is given by $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\alpha}{z} = 2$. 10

13. Show that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$. 10