EX/ET/T/323/2017

B. ETCE 3RD YEAR 2ND SEMESTER EXAMINATION 2017

DIGITAL SIGNAL PROCESSING Time: Three Hours Full Marks: 100

Answer any ***Five*** questions.
All questions carry equal marks.

1. a) Explain how a sequence could be expressed as a sum of scaled and delayed unit-samples. 4

   b) Derive the expression for the output sequence of an LTI system in terms of the input sequence and impulse response. 8

   c) Determine analytically the output sequence y(n) if the input sequence and the impulse response are given by

$$x(n) = \begin{cases} n/3 & 0 \le n \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$ 8

2. a) Determine graphically the response from an LTI system having impulse response h(n) = { 8, 7, 6, 5, 4, 3, 2, 1} to the input sequence

   x(n) = { 8, 8, 8, 8, 8} 10

   b) Show that the system y(n) = a n x(n) is time-variant. 2

c) The following input-output pairs have been observed during the operation of a time-invariant system.

$$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xrightarrow{H} y_1(n) = \{\underset{\uparrow}{0}, 1, 2\}$$

$$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xrightarrow{H} y_2(n) = \{\underset{\uparrow}{0}, 1, 0, 2\}$$

$$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xrightarrow{H} y_3(n) = \{\underset{\uparrow}{1}, 2, 2\}$$

Can you draw any conclusion regarding the linearity of the system? What is the impulse response of the system? 6 + 2

3. a) Justify the statement:

"If we have 'L' LTI systems in cascade with impulse responses $h_1(n)$, $h_2(n)$, $h_3(n)$,.....$h_L(n)$, there is an equivalent LTI system having an impulse response which is equal to the (L – 1) – fold convolution of the impulse responses." 8

b) An LTI system is composed of three systems having impulse responses as

$h_1(n) = 3\delta(n) + \delta(n-1)$; $h_2(n) = 2\delta(n) + \delta(n-2)$
$h_3(n) = \delta(n) - 3\delta(n-1) + 7\delta(n-4) + 6\delta(n-6)$.

Determine the overall impulse response if the system with impulse response of $h_3(n)$ is connected in parallel with the cascade connection of the two systems having impulse responses of $h_1(n)$ and $h_2(n)$. 7

c) Consider an LTI system with impulse response h(n). If the input x(n) is a periodic sequence with a period N, show that the output y(n) is also a periodic sequence with the period N. 5

4. a) Derive the necessary and sufficient condition for the causality of an LTI system. 8

b) Check for the BIBO stability of the following systems: 4 X 2

i) $y(n) = e^{-x(n)}$

ii) $y(n) = ax(n+1) + bx(n-1)$

iii) $y(n) = \text{average of } [x(n+1),\ x(n),\ x(n-1)]$

iv) $y(n) = ax(n) + bx^2(n-1)$

c) Determine the range of values of the parameter 'μ' for which the LTI system with impulse response $h(n) = \mu^n \cdot u(n)$ is stable. 4

5. a) Plot the values of the 8-point Twiddle factor and hence conclude about the general properties of it. 8 + 2

b) Show that $W_N^{N/4} = -j$, $W_N^{3N/4} = j$ and $W_N^{N} = 1$ where the symbols have their usual significance. 4 + 4 +2

6. a) Differentiate between linear and circular convolutions. 4

b) Find graphically the circular convolution of the sequences $x_1(n) = \{\underset{\uparrow}{1}, 3, 2, -1\}$ and $x_2(n) = \{\underset{\uparrow}{2}, 1, 0, -1\}$. 10

c) Verify the result obtained in Part 'b' by DFT/IDFT method. 6

7. a) Show how the N-point DFT could be expressed in terms of the N/2 – point DFTs of the decimated time-domain sequences. What is the computational advantage of such decimation? 4 + 2

b) Differentiate between the Butterfly structures of DIT and DIF FFT algorithms. 4

c) Derive the signal-flow graph for computing the 8-point DFT of the sequence $x(n) = \cos(n\pi/2)$ using DIT FFT algorithm. 10

8. a) Calculate the number of stages and also the number of Butterflies in each stage for the computation of 4096-point DFT by FFT algorithm. Calculate also the complex multiplication speed improvement factor over direct computation. 2 + 2 +2

b) Compute the 8-point DFT of the sequence $x(n) = n + 1$ using the signal-flow graph of DIF FFT algorithm. 10

c) Explain the concept of "computations in one place" with respect to the signal-flow graph of Part 'b'. 4