Ref. No.: Ex/ET/T/324/2017

B.E. ELECTRONICS AND TELECOMMUNICATION ENGINEERING

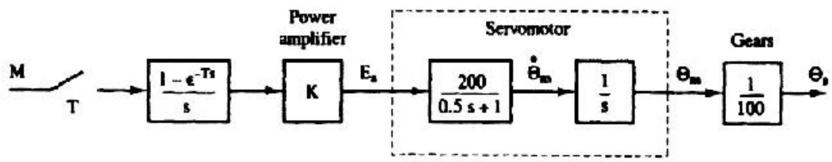
THIRD YEAR SECOND SEMESTER 2017

Subject: DIGITAL CONTROL SYSTEMS    Time: 3 Hours    Full Marks: 100

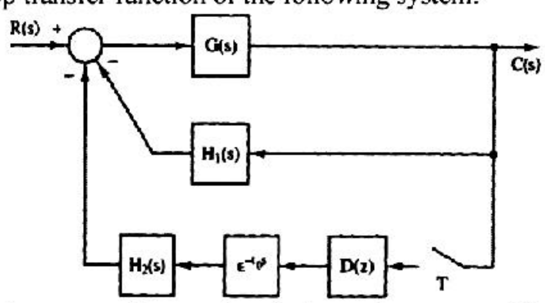
Answer ANY FOUR.

All parts of the same question must be answered at one place only.

1 (a) Derive the open loop transfer function of the following control system of one joint of a robot with $K = 2.4$, $T = 0.1$ s, $E_a(s)$ as the servo motor input voltage, $\theta_m(s)$ as the motor shaft angle and $\theta_a(s)$ as the arm angle. 10



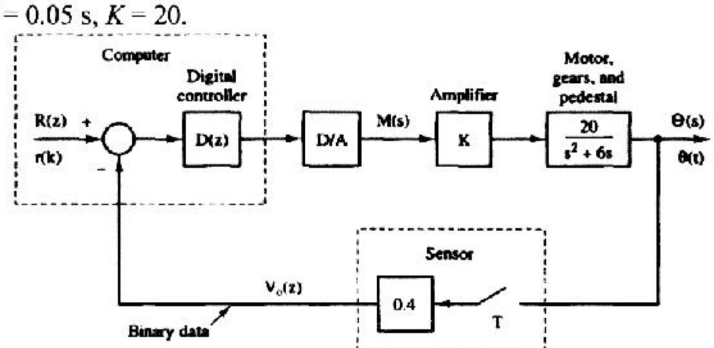

(b) Find the closed loop transfer function of the following system. 8




(c) Determine the maximum overshoot and peak-time of a system with closed-loop transfer function as 7

$$\frac{C(z)}{R(z)} = \frac{0.4986(z+0.7453)}{z^2 - 1.262z + 0.5235}.$$

2. (a) Evaluate the closed loop transfer function of the following antenna control system with $D(z) = 1$, $T = 0.05$ s, $K = 20$. 10




(b) Given the closed loop transfer function of a digital control system 10

$$\frac{C(z)}{R(z)} = \frac{T^2\left(K_P z^2 + K_I Tz + K_I T - K_P\right)}{Az^3 + Bz^2 + Cz + D}$$

where, $J_v = 41822$,

$A = 2J_v$,

$B = T^2K_P + 2K_RT - 6J_v$,

$C = 6J_v - 4K_RT + T^3K_I$,

$D = 2K_RT + T^3K_I - 2J_v - T^2K_P$

Find the values of $K_P$, $K_I$ and $K_R$ as functions of $T$ so that the step response $c(kT)$ follows

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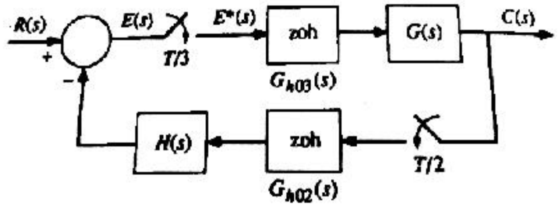
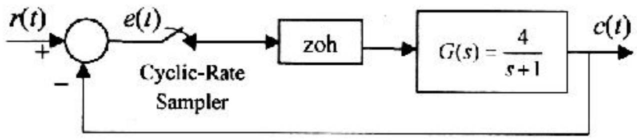
the step input in a minimum number of sampling periods. What is the maximum overshoot $c(kT)_{max}$?

(c) Draw the Direct Form-II realization of a *PID* controller using trapezoidal rule for numerical integration. 5

3. (a) Determine the closed loop transfer function for the following multi-rate control system. 12

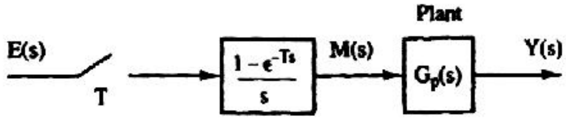



(b) Find the state equations of the following closed loop system with the cyclic sampler operating at $kT$, $kT + T_1$, where $T = 1$ s and $T_1 = 0.25$ s. 13




4. Consider the following system with the behavior of the plant described by the first order differential equation

$$\frac{d^2y(t)}{dt^2} + 0.15\frac{dy(t)}{dt} + 0.005y(t) = 0.1m(t).$$




(a) Draw a continuous-time simulation diagram for $G_p(s)$ and give the state equations. 7

(b) Use the state-variable model of part (b) to find a discrete state model for the entire system. 10

(c) Obtain the system transfer function from the derived discrete state-variable model. 8

5. (a) Consider a system with state equation given by

$$\vec{x}(k+1) = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}\vec{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k), \ \ \vec{y}(k) = \begin{bmatrix} -2 & 1 \end{bmatrix}\vec{x}(k).$$

A similarity transformation of the system described above yields

$$\vec{w}(k+1) = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}\vec{w}(k) + \mathbf{B_w}u(k), \ \ \vec{y}(k) = \mathbf{C_w}\vec{w}(k).$$

(i) Determine $d_1$ and $d_2$. 5

(ii) Find a similarity transformation that results in $\mathbf{A_w}$ given. 5

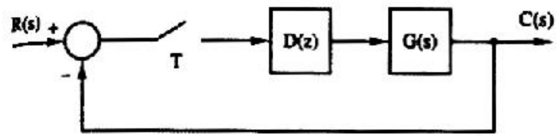
(iii) Find $\mathbf{B_w}$ and $\mathbf{C_w}$. 5

(b) Show that for the similarity transformation 5

$$\mathbf{C}\left[z\mathbf{I} - \mathbf{A}\right]^{-1}\mathbf{B} + \mathbf{D} = \mathbf{C_w}\left[z\mathbf{I} - \mathbf{A_w}\right]^{-1}\mathbf{B_w} + \mathbf{D_w}.$$

(c) What is bilinear transformation? 5

6. (a) Design a digital controller $D(z)$ as given in Fig. 7 to attain a steady state error less than 0.01 for unit ramp input and to ensure stability of the entire system with $G(s) = \frac{1-\exp(-Ts)}{s(s+1)}$ and $T$=0.1 sec. 8




(b) Explain the effect of addition of open loop poles to the system stability with the help of root locus technique. 5

(c) State and prove Nyquist stability criterion for digital control system. Using Nyquist stability criteria, comment on stability of a closed loop system with open loop transfer function 12

$$\overline{GH}(z) = \frac{0.01kz}{(z-1)^2(z-0.905)}.$$

7. (a) For a plant described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

find the gain matrix $K$ required to realize the closed loop characteristic equation with zeros providing a damping ratio of 0.46 and a time constant of 0.5 s.

(b) Find out the oscillating frequency for the marginal stability of a unity feedback discrete-time control system with open loop transfer function 6

$$G(z) = \frac{0.368z + 0.264}{z^2 - 1.368z + 0.368} K.$$

(c) Derive the transfer function of a reduced order state observer. 9

8. (a) State Lyapunov stability theorem for linear time-invariant discrete system. 5

(b) Consider a linear digital control system described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k).$$

Find the optimal control $u^o(k)$ so that the Lyapunov function $V(\vec{x}) = \vec{x}^T(k)\mathbf{P}\vec{x}(k)$ is minimized where $\mathbf{P}$ is a positive definite solution of $\mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} = -\mathbf{I}$.

(c) Given a first order plant described by $x(k+1) = 0.9x(k) + 0.1u(k)$ with the cost function 10

$$J_3 = \sum_{k=0}^{3} \left( x^2(k) + 5u^2(k) \right)$$

calculate the feedback gains required to minimize the cost function.

9. (a) Derive the mean and variance of quantization noise. 10

(b) Determine the quantization noise at the output of a parallel digital controller due to multiplication. 10

(c) The incoming error signal has saturation to threshold ratio of 250. The allowable noise figure is 40 dB. Find the word length of the ADC. 5