b) State and prove Euler's theorem on homogeneous functions of two variables. 5

11. a) If $z = f(x, y)$ and $x = e^{x} + e^{-v}$, $y = e^{-u} - e^{v}$ then prove that

$$\frac{\partial z}{\partial x} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$ 5

b) If $\tan u = \frac{x^3 + y^3}{x - y}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$ 5

12. a) If $x = r\cos\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ then show that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2\sin\theta$ 5

b) Examine the equation $x^2 + x^3y + y^2 = 0$ for the existence of unique implicit function near the point (0, 0). 5

13. a) Find and classify the extreme values (if any) of the function defined by $f(x, y) = x^3 + y^3 - 3x - 12y + 20.$ 5

b) Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225,\ z = 0.$ 5

14. Find the maxima and minima of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $z = x + y.$ 10

Bachelor of Electronics & Tele-communication Engineering Examination, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - IG

Time : Three hours Full Marks : 100

(50 marks for each Group)

Use a separate Answer-Script for each Group.

GROUP - A

Answer ***any five*** questions.

1. a) If $y = (x^2 - 1)^n$ then prove that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$ 5

b) State Cauchy's mean value theorem. Show that there exists $c(a \angle c \angle b)$ such that c is the geometric mean between a and b by using Cauchy's mean value theorem for the functions $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in $[a, b]$. 5

2. a) State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$. 5

b) Show that $\frac{x}{1+x} < \log(1+x) < x$ for all $x > 0$. 5

[Turn over

3. a) Find the maximum value of $x^{\frac{1}{x}}$. 5

b) Show that all rectangles of given area, the square has the smallest perimeter. 5

4. a) Determine $\lim_{x\to 0}\frac{e^{x}-e^{-x}-2\log(1+x)}{x\sin x}$ 5

b) Find all the asymptotes of the curve

$y^3-5xy^2+8x^2y-4x^3-3y^2+9xy-6x^2+2y-2x=1$ 5

5. Prove that $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}\cdots$

Justify the convergence of the series. 10

6. a) Prove that $\lim_{x\to\infty}\frac{\{(n+1)(n+2)\cdots(2n)\}^{\frac{1}{n}}}{x}=\frac{4}{e}$ 5

b) Examine the convergence of the series

$x+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1.3}{2.4}\cdot\frac{x^5}{5}+\cdots(x>0)$. 5

7. a) Prove that the sequence $\{f_n(x)\}$ of functions where $f_n(x)=\frac{x}{n+x^2}, x\in[0,1]$ is uniformly convergent on [0, 1]. 5

b) Prove that the series $\sum\frac{\cos nx}{n(n+1)}$ is uniformly convergent for all real x. 5

GROUP - B

Answer ***any five*** questions.

8. Evaluate $\lim_{x\to 0}\lim_{y\to 0} f(x,y)$, $\lim_{y\to 0}\lim_{x\to 0} f(x,y)$ and $\lim_{(x,y)\to(0,0)} f(x,y)$, if they exist, for the following functions

i) $f(x,y)=\frac{x^3+y^3}{x-y},\ x\neq y$
$\quad = 0 \quad , x=y$ 5

ii) $f(x,y)=y\sin\frac{1}{x}+\frac{xy}{x^2+y^2},\ x\neq 0$
$\quad = 0 \quad , x=0$ 5

9. a) Show that $f(x,y,z)=\frac{x^2+y^2-2z^2}{x^2+y^2+z^2}$ is not continuous at the origin (0, 0, 0). 5

b) Find the directional derivative of the function $f(x,y)=e^x\cos y$ at the point (0, 0) in the direction of $(\sqrt{3},1)$. 5

10. a) If $f(x,y)=\begin{cases} xy\frac{x^2-y^2}{x^2+y^2} & \text{when } (x,y)\neq(0,0) \\ 0 & \text{when } (x,y)=(0,0)\end{cases}$

then show that $f_{xy}(0,0)\neq f_{yx}(0,0)$. 5