

Ex/ET/MATH/T/114/2017(S)
 B.E.T.C.E. Examination, 2017
 (1ST YR, 1ST SEM , SUPPLY)
 MATHEMATICS
 PAPER - II G

Full Marks : 100

Time: Three hours

Answer any 10 questions . $10 \times 10 = 100$

1. What is an analytic function?
State and prove Cauchy-Riemann Equation.
2. State and prove Cauchy's Inequality formula.
3. (a) Define with examples of regular point, singular point, isolated singularity and removal singularity.
(b) Evaluate the residues of $f(z)$ where

$$f(z) = \frac{e^z}{z^2(z^2 + 9)} \text{ at } z = 0.$$

4. Find the analytic function $f(z) = u + iv$ of which the complex part is

$$v = 6xy - 5x + 3.$$

5. Find the value of the

$$\oint_C \frac{dz}{1 + z^2},$$

where C is the contour

$$\left| z - \frac{i}{2} \right| = 1.$$

6. (a) If $\vec{r} \times d\vec{r} = 0$, show that \vec{r} is a constant vector.

(b) Find the angle between two vectors

$$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k} \text{ and } \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}.$$

7. Define solenoidal vector. Find a so that the vector

$$\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + az)\vec{k}$$

is solenoidal.

8. Solve the equation using Z-Transformation

$$f(n + 2) - 3f(n + 1) + 2f(n) = 0, \text{ given : } f(0) = 1, f(1) = 2$$

9. Solve the equation using Laplace Transformation

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 2x = 0 \text{ given : } x(0) = x'(0) = 1.$$

10. Find the Fourier Transformation of $Ne^{-\alpha t^2}$.

11. Show that $\vec{F} = (\sin y + z)\vec{i} + (x \cos y - z)\vec{j} + (x - y)\vec{k}$ is a conservative field and find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$.

12. Find the Fourier series of the function

$$f(x) = x \sin x, \text{ when } -\pi < x < \pi$$

Deduce also

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$$