EX/ET/MATH/T/124/2017

B. ETCE 1st year 2nd sem 2017
Mathematics-IV G

Time : Three hours Full Marks : 100

( 50 marks for each Group )

Use a separate Answer-Script for each Group

## Group-A
(Answer any five questions)

1. (a) Use Cramer's rule to solve the system of equation : 5

$$\begin{aligned} 3x + 2y &= 3z + 1 \\ 3x + 2z &= 8 - 5y \\ 3z - 1 &= x - 2y \end{aligned}$$

(b) Compute $AB$ using block multiplication where 5

$$A = \begin{pmatrix} 1 & 2 & . & 1 & 0 \\ -3 & 4 & . & 0 & 1 \\ . & . & . & . & . \\ 0 & 0 & . & 2 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & . & 2 \\ 0 & 1 & . & 3 \\ . & . & . & . \\ 2 & 3 & . & 4 \\ 3 & -2 & . & 1 \end{pmatrix}$$

2. (a) Write the following system of equations in the form $AX = B$ and then then solve it by finding $A^{-1}B$ 5

$$\begin{aligned} 2x - y + 3z &= 2 \\ y - 4z &= 5 \\ 2x + y - 2z &= 7 \end{aligned}$$

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2.(b) Find the rank of the following matrix 5

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & 2 \\ 1 & 2 & -2 & -4 & 3 \end{pmatrix}$$

3. (a) Justify with reason whether the following mappings from $R^3$ to $R^3$ are linear or not

(i) $T(x, y, z) = (2x, 3y, x + y)$
(ii) $S(x, y, z) = (2x^2, 3y, 4z)$ 5

(b) Let $T$ be the linear mapping on $R^2$ defined by $T(x, y) = (-y, x)$. What is the matrix of $T$ w.r.t. the ordered basis $\{(1, 2), (1, -1)\}$? 5

4. (a) Find the eigenvalues and eigenvectors of the following matrix

$$\begin{pmatrix} 1 & -4 & -1 \\ 3 & 2 & 3 \\ 1 & 1 & 3 \end{pmatrix}$$

5

(b) Let 5

$$A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{pmatrix}$$

Then find a non-singular matrix $P$ such that $P^{-1}AP = D$, where $D$ is a diagonal matrix consisting of the eigenvalues of $A$.

5. (a) Verify Cayley-Hamilton's theorem for the matrix $A$ where 5

$$A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & 3 & 3 \end{pmatrix}$$

5.(b) Compute $e^A$ by diagonalizing the matrix

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$$

6. (a) If $\sum a_n$ with $a_n > 0$ is convergent then is $\sum {a_n}^2$ is always convergent? Either prove it or give a counter example. 5

(b) Test the convergence of the following series 3+3

(i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$ (ii) (ii) $\sum_{n=1}^{\infty} \frac{(2n)!}{n^n}$

Group-B

( Answer any five questions )

**(Use separate answerscript for this group)**

1. (a) Determine the order, degree, the unknown function and the dependent variable of $(\frac{d^2x}{dt^2})^2 + \frac{d^2x}{dt^2} + t(\frac{dx}{dt})^3 = 0$ 3

   (b) Solve $(D^4 - n^4)y = 0$ completely.
   Now if $Dy = y = 0$ when $x = 0$ and $x = l$, prove that
   $y = A(cos\ nx - cosh\ nx) + B(sin\ nx - sinh\ nx)$ and $cos\ nl\ cosh\ nl = 1.$ 7

2. (a) Test whether the equation $(x^3 + 4xy)dx + (2x^2 + 2y)dy = 0$ is exact.
   Hence solve it. 5

   (b) Solve: $y^2 + (x - \frac{1}{y})\frac{dy}{dx} = 0.$ 5

3. (a) Solve: $\frac{dy}{dx} - \frac{tan\ y}{1+x} = (1 + x)e^x\ sec\ y.$ 5

   (b) Solve: $(x^2y^3 + 2xy)dy = dx.$ 5

4. (a) Find the ordinary point and regular singular point of
   $x^2(x-2)^2\frac{d^2y}{dx^2} + 2(x-2)\frac{dy}{dx} + (x+1)y = 0$ 2

   (b) Use the method of Frobenius to find solutions of the differential equation
   $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$
   in some interval $0 < x < R$ . 8

5. (a) Solve: $(D^3 - 2D^2 - 5D + 6)y = (e^{2x} + 3)^2 + e^{3x}\ coshx.$ 5

   (b) Solve: $\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 4y = e^{2x} + e^x + 3e^{-x}.$ 5

6. (a) To find a particular integral of
   $(D^2 + 4)y = xsin^2x.$ 6

   (b) Test whether the equation
   $(cosy + ycosx)dx + (sinx - xsiny)dy = 0$ is exact.
   Hence solve it. 4

7. (a) Solve the following system
   $\frac{dx_1}{dt} = -5x_1 + x_2$
   $\frac{dx_2}{dt} = 4x_1 - 2x_2$ with $x_1(0) = 1,\ x_2(0) = 2.$
   Hence sketch the phase portrait for the system. $5 + 5$