Ex/ET/Math/T/123/2017

Bachelor of Engineering in Tele-communication Engineering Examination, 2017

(1st Year, 2nd Semester)

Mathematics - IIIG

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

(Unexplained Notations and symbols have their usual meanings)

PART - I

Answer ***any five*** questions :

1. a) Suppose X is a nonempty set and F is a field. Suppose Map (X, F) denotes the set of all maps from X to F.

a) Prove that Map (X, F) is a vector space over F where the operations are defined as follows :

$(f+g)(x) := f(x)+g(x)$

$(\alpha f)(x) := \alpha f(x)$ for all $f, g \in$ Map(X, F),

for all $x \in X$, for all $\alpha \in F$.

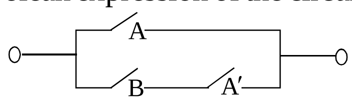
b) Suppose $X\{a,b,c\}$ and $F = \mathbb{R}$ the field of real numbers. Prove that $\{f_a, f_b, f_c\}$ is a basis of the vector space Map $(X, \mathbb{R})$ where for $x \in X, f_x$ is defined by

$$f_x : X \to F, f_x(y) = 1 \text{ if } x = y$$
$$= 0 \text{ if } x \neq y$$

[Turn over

c) Prove that the norm induced by an inner product satisfies the parallelogram law. 4+3+3

2\. a) Give a Boolean expression of the circuit



b) A hall light is controlled by two switches one upstairs and one downstairs. Design a circuits so that the light can be switched on or off from the upstairs or the downstairs.

c) Consider the Boolean Expression

$AV(B' \vee C) \wedge (A \vee B \vee C) \ldots \circledast$

i) Define the truth table of the Boolean function

$f : \{0;1\} \times \{0,1\} \times \{0,1\} \rightarrow \{0,1\}$

where f is defined by the Boolean Expression $\circledast$

ii) Find the disjunctive normal form of the Boolean Expression $\circledast$

iii) Draw two circuits representing the Boolean Expression $\circledast$ 1+1+(2+3+3)

3\. a) Define a Boolean lattice

Let L be a Boolean lattice.

Prove that i) de Morgan's laws hold in L,

ii) $a \wedge b' = 0$ if and if $a \leq b$ for all $a, b \in L$.

12\. a) In a Markov chain having state space S={0, 1}, let X_n denote the state of the machine at time n.

Given that $P(X_{n+1} = 1 | X_n = 0) = p$,

$P(X_{n+1} = 0 | X_n = 1) = q$, $P(X_o = 0) = \pi_o(0)$.

Find $P(X_n = 0)$ and $P(X_n = 1)$.

b) Write down chapman-Kolmogorov equation, hence derive its Forward and Backward equation. 6+4

5. a) Suppose G = {a, b, c}.

Prove that any two groups with G as the underlying set are isomorphic

(Hint : Prove that each is isomorphic to $\mathbb{Z}_3$)

b) Find the elements a in the group $\mathbb{Z}_{12}$ such $\mathbb{Z}_{12}$ that is generated by a.

c) Suppose X is a nonempty set. Define suitable + (addition) and $\cdot$ (miltiplication) on P(X), the power set of X such that (P(X), +, $\cdot$) becomes a ring (verify ring axioms).

Show that it is a Boolean ring. 3+3+4

6. a) Draw the Hasse diaram of the following posets.

i) P = {1, 2, 3, 12, 18, 0} with divisibility relation,

ii) P = the set of subgroups of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ with set inclusion. Discuss also for each of the above posets the boundedness, lattice structure, distributive properties, complements, Boolean structure.

b) Why are the groups $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_4$ not isomorphic ?

8+2

PART - II

Use a separate Answer-Scripts)

Answer ***any five*** questions.

6. a) The chance that a doctor will diagnose a certain disease correctly is 60%. The chance that a patient will die under his teatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of the doctor who had the disease died. What is the chance that his disease was diagnosed correctly ?

b) If A, B are two independent events in a random experiment, show that

i) A^C and B,

ii) A^C and B^C are independent. 6+4

7. a) A card is drawn from a full pack and replaced 260 times. Find the probability of obtaining queen of hearts 4 times.

b) Find mean and variance of the Binomial (n, p) distribution, where n and p denote the number of trials and probability of success respectively. 4+6

8. a) Show athat function f(x) given by

$$f(x)\begin{cases} x & \text{for } 0 < x < 1 \\ k - x & \text{for } 1 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

is a probability density function for a suitable value of the constant K. Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$.

b) If X is normal (μ, σ) vriate, prove that $P(a < X < b) = \phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)$ and $P(|X-\mu| > a\sigma) = 2[1-\phi(a)]$ where $\phi(x)$ denotes the standard normal distribution function. 6+4

9. A point P is taken at random on a line segment AB of length 2a. Find the probability that the area of the rectangle consisting of the sides AP and PB will exceed $\frac{1}{2}a^2$. 10

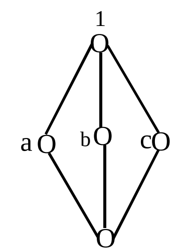
10. Suppose that the travel time from your home to your office is normally distributed with mean 40 minutes and standard deviation 7 minutes. If you want to be 95 percent certain that you will not be late for an office appointment at 1 P.M., what is the latest time that you should leave home ? Given that $\phi(1{\cdot}645) = 0{\cdot}95$; where $\phi(x)$ denotes the area under the standard normal curve to the left of x. 10

11. Derive the probability for the steady-state distribution that there are n units in the system for the queueing model (Birth and Death Model) : $(M/M/1 : \infty/FIFO)$ 10

b) If L is a finite Boolean lattice then what is the number of element of L ?

c) If an element of a distributive lattice has a complement then it is unique – Explain !

d) What conditions are lacking in the poset



for it to be a Boolean lattice ? 5+1+2+2

4. a) Suppose $\mathbb{R}^3$ is equipped with the standard inner product. Find an orthonormal basisi of the subspace V of $\mathbb{R}^3$ spanned by {(1, 0, 1), (2, 1, 3}) Also find the best approximation of (1, 2, 3) in V.

b) Define an inner product < , > on $\mathbb{R}^2$ such that <(1, 0), (0, 1)> = 2 and verify that it is an inner product.

c) Prove that the matrices $E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $E_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$, $E_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ form a basis of the vector space $M_2(\mathbb{R})$ of all 2 x 2 real matrices. 5+3+2