EX/ EE/MATH/5/T/113//2017(S)

BACHELOR OF ENGINEERING IN ELECTRICAL ENGINEERING (EVENING) EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics-IIF

Time: Three hours | Full Marks: 100

(Symbols and notations have their usual meanings)
Answer any *five* questions

1. a) Show by vector method that the join of the middle points of two sides of a triangle is parallel to the third side and is half of its length. 5

   b) Find the equations of the tangent plane to the surface xyz = 4 at the point $\hat{\imath}+2\hat{\jmath}+2\hat{k}$. 5

   c) Prove by vector method $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ in a triangle ABC. 5

   d) Show that $\vec{\nabla}\times(\vec{\nabla}\times\vec{F})=\vec{\nabla}(\vec{\nabla}.\vec{F})-\vec{\nabla}^2\vec{F}$. 5

2. a) Show that $\vec{\nabla}\times(\vec{\nabla}\times\overrightarrow{F})=\vec{0}$. 4

   b) Show that the vector $\vec{F}=x^2\hat{\imath}+y^2\hat{\jmath}+z^2\hat{k}$ is a conservative force field. Show that $\vec{F}$ can be expressed as the gradient of some scalarpoint function φ. 8

   c) Verify Stoke's theorem for the vector function $\vec{F}=(x^2-y^2)\hat{\imath}+2x\hat{\jmath}$ around the rectangle bounded by straight lines x=0, x=a, y=0, y=b. 8

3. a) Solve: 2x5=10

   i. $x\,dx+y\,dy=xdy-ydx$
   ii. $x^2y\,dy-\left(xy^2-e^{\frac{1}{x^3}}\right)dx=0$

   b) Solve the following Euler-Cauchy differential equation 10

$$(x+2)^2\frac{d^2y}{dx^2}-4(x+2)\frac{dy}{dx}+6y=x$$

4. a) Solve the following: 2x5=10

   i. $y(2xy+e^x)dx-e^x\,dy=0$
   ii. $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$

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b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ 10

5. Solve: 4x5=20

a) $(D^2 + 4)y = \sin 2x$

b) $xy\mathrm{d}x + (2x^2 + 3y^2 - 20)\,\mathrm{d}y = 0$

c) $x\cos(\frac{y}{x})(ydx + xdy) = y\sin(\frac{y}{x})(xdy - y\,dx)$

d) $(D^2 - 5D + 6)y = x^2$

6. a) Solve the equations by Cramer's rule

$$x + 2y + 3z = 6,$$
$$2x + 4y + z = 7,$$
$$3x + 2y + 9z = 14.$$

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b) Eliminate $l,\ m,\ n$ from the equations $al + cm + bn = 0,\ cl + bm + an = 0,\ bl + am + cn = 0$ and express the result in the simplest form. 7

c) Find the rank of the matrix $\begin{pmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 1 & 4 & 4 \end{pmatrix}$ 6

7. a) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

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Hence obtain A^{-1}.

b) Find the eigenvalues and the corresponding eigen vectors of the matrix

$$\begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

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c) If A be a square matrix, then show that $A + A^t$ is symmetric and $A - A^t$ is skew-symmetric. 4

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