Ref. No *Ex/EE/T/423A/2017*

**Bachelor of Electrical Engineering Examination, 2017**
**4th Year, 2nd Semester**

# Advanced Control Theory

Time: Three Hours; Full Marks: 100

**Answer both parts on the same answer script**

## Part-I

**Answer any *three* questions from this part (all questions carry equal marks)**

*Two* marks for neat and well-organized answerscript

1. A nonlinear block exhibits a deadzone of +/- 2 V, unit slope and a saturation at +/- 24 V. **4+2+4+3+3**
   a) Describe the nonlinearity algebraically.
   b) Does this nonlinearity have memory? Explain.
   c) If a sinusoidal excitation of 24 V RMS is applied to the system, obtain the algebraic expression for the output waveform.
   d) Sketch the output waveform.
   e) Obtain the Fourier expansion of the output waveform up to 3 significant terms.

2. a) Explain what is meant by "Equilibrium Point". Show that an inverted pendulum has one stable and one unstable equilibrium point. (Derivation of the dynamic model is not necessary.) **4+(4+4+4)**
   b) A nonlinear system is expressed as follows:

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 + x_2\left(1 - 3x_1^2 - 2x_2^2\right).$$

   i. Determine the equilibrium point(s) of the above system.
   ii. Linearize the above system about its equilibrium point(s).
   iii. Comment about the asymptotic stability of the system at $\boldsymbol{x}=0$.

3. a) State Lyapunov's <u>2nd</u> theorem. Briefly describe how this theorem may be used to determine the stability of a nonlinear dynamic system. What are its limitations? **6+10**
   b) The dynamics of an unforced nonlinear system is described by
   $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - x_1\left(x_1^2 + x_2^2\right) \\ -x_1 - x_2\left(x_1^2 + x_2^2\right) \end{bmatrix}$. Using the function $V = \frac{1}{2}\left(x_2^2 + x_1^2\right)$ as the Lyapunov function, investigate the stability of the system about its equilibrium point at the origin.

[ Turn over

4. a) Enumerate the advantages and disadvantages of on-off control. **2+4+10**
   b) With schematic diagrams explain how an on-off type temperature control system functions. State the necessary controller characteristics.
   c) (i) Sketch the time response of the above first order plant with finite delay, Proportional-derivative type on-off control.
   (ii) Mark on-time and off-time, maximum temperature and minimum temperatures on this sketch
   (iii) Derive approximate expressions for on-time, off-time, duty cycle and maximum temperature.

5. a) What is a phase plane plot? How does it help stability analysis of nonlinear systems? Explain the meaning and use of isoclines for this plot. **4+6+6**
   b) With suitable phase plane diagram discuss how the stability of standard second order system with different pole locations may be analyzed by their phase portraits.
   c) A satellite attitude control system has forward-reverse type of thrusters and a controller with proportional plus derivative control with dead zone. With the help of a phase plane plot investigate the stability of the system.

## Part II

**<u>Answer any *three* questions from this part (all questions carry equal marks)</u>**

*Two* marks for neat and well-organized answerscript

6. a) Explain the difference between the terms 'Structured uncertainty' and 'Unstructured uncertainty. **4+12**
   b) Check for the robust stability of the system whose characteristic polynomial is given by

$$p_6 s^6 + p_5 s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 = 0,$$

where

$$p_6 \in [1, 1],\ p_5 \in [20, 30],\ p_4 \in [120, 140],\ p_3 \in [615, 642],$$
$$p_2 \in [1350, 1360],\ p_1 \in [11, 15] \text{ and } p_0 \in [650, 660].$$

7. **(2+4)+5 +5**
   a) A process plant given by $G_1(s) = \dfrac{10}{(s+1)(0.01s+1)}$ is modeled by using the transfer function $G_2(s) = \dfrac{10}{s+1}$. Compare (i) the open loop unit step responses, and (ii) the frequency responses of the plant and its model.

b) Given a transfer function $G(s)=\dfrac{12}{(s+1)(s+2)^2(s+3)}$. Find $\|G\|_2$.

c) For the system with transfer function $G(s)=\dfrac{0.5s+1}{0.2s+1}$, find $\|G\|_\infty$.

8. a) What is an observer? What are its uses? Explain with the help of a block diagram. **6+10**

b) Design a reduced order observer for observing the second state variable for the following continuous time system so that the observer pole is located at - 8,

$$\dot{\mathbf{x}}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ 0.01\end{bmatrix}u\ ; \quad y=[1\ \ 0]\,\mathbf{x}\,.$$

9. a) What is optimal control? Explain the meaning of the term 'quadratic performance index'. **4+10+2**

b) Explain what is meant by the following terms giving an example in each case:

(i) The tracking control problem

(ii) The regulator control problem

(iii)The terminal control problem

(iv) The minimum-time control problem

(v) The minimum energy control problem

c) State the expression for the Hamilton-Jacobi equation clearly mentioning all notations used.

10 A regulator contains a plant described by **6+6+4**

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}0 & 1\\ -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}u\ ; \quad y=[1\ \ 0]x$$

and has the performance index

$$J=\int_0^\infty\left[x^T\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}x+u^2\right]dt\,.$$

Determine

a) the Riccati matrix P

b) the optimal control law

c) the closed loop eigenvalues.