Ref No: Ex/EE/T/321/2017

B.E. ELECTRICAL ENGINEERING THIRD YEAR SECOND SEMESTER EXAMINATION, 2017

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Full Marks 100

Time: Three hours (50 marks for each part)

Use a separate Answer-Script for each part

No. of Questions	PART- I	Marks
	Answer any *THREE* questions **Two marks reserved for neatness and well organized answers.**	
1. (a)	A store carries flash drives with either 1 GB, 2 GB, 4 GB, 8 GB, or 16 GB of memory. The accompanying table gives the probability distribution of Y = the amount of memory in a purchased drive: x(GB): 1, 2, 4, 8, 16 P(x): 0.05, 0.10, 0.35, 0.40, 0.10 Determine the following: (i) Probability of a purchasing a drive having a memory of at least 3.5 GB. (ii) Probability of purchasing a drive with a memory in the range 2.3 to 7.8 GB. (iii) The statistical mean and the variance of the memory in the purchased drives.	6
(b)	In proof testing of circuit boards, the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 200 diodes. (i) How many diodes would you expect to fail, and what is the standard deviation of the number that are expected to fail? *Derive the expressions used.* (ii) What is the (approximate) probability that at least four diodes will fail on a randomly selected board? (iii) If five boards are shipped to a particular customer, how likely is it that at least four of them will work properly? (A board works properly only if all its diodes work).	10

[Turn over

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No. of Questions	PART I	Marks
2. (a)	Prove that the variance of a sum of Independent Random Variables is equal to the sum of their variances.	4
(b)	From the notion of the Joint Central Moment, establish mathematically the notion of covariance. Also prove that $\lvert C_{X,Y}\rvert \leq \sigma_X \, \sigma_Y$, where all the symbols carry their usual meanings.	6
(c)	State, under which conditions, the Poisson Distribution can be derived as the limit of the Binomial Distribution. Hence, derive the Poisson Distribution as the limit of the Binomial Distribution.	6
3. (a)	Given a non-negative integer-valued discrete random variable X with p{X=k} = P_k, show that the Probability Generating Function can be called the Z-transform of X. Hence, derive the expressions of mean, variance and the k^{th} moment.	6
(b)	The amplitudes of two signals *X* and *Y* have joint *pdf* $f(x,y) = e^{-x/2} y e^{-y^2}$ for $x>0$, $y>0$ (a) Find the joint cumulative distribution function (cdf). (b) Find the marginal pdfs. (c) The correlation of *X* and *Y*, if they are statistically independent.	10
4. (a)	Explain in brief the concept of random process. Cite any suitable practical example of a deterministic random process. Distinguish between 'Strict-sense Stationarity' and 'Wide-sense Stationarity' of random processes.	5+2+3

	PART- I	
(b)	A sample realization of a random process having an autocorrelation function $R_X(\tau) = 16 + 16e^{-2\|\tau\|}$ is the input to a linear system having an impulse response of $h(t) = \delta(t) + 2e^{-2t}u(t)$. Find the (a) mean value of the output. b) mean-square value of the output. (c) variance of the output. OR *X(t)* is a wide-sense stationary random process with an autocorrelation function $R_X(\tau) = 16e^{-a\|\tau\|}$; $a > 0$ is a constant. Consider a random process $Y(t) = X(t)\text{Cos}(\omega_o t + \Theta)$, where ω_0 is a constant, and Θ is a random variable uniformly distributed in the range $(-\pi, +\pi)$. *X(t)* and Θ are statistically independent. Determine the relation between $R_Y(\tau)$ and $R_X(\tau)$. Can you comment on whether or not *Y(t)* is a wide-sense stationary random process?	6
5.	Write short notes on any <u>two</u> of the following. (a) White Noise. (b) Markov's inequality and Chebyshev's inequality. (c) Power spectral density of random processes. --	8+8

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Bachelor of Electrical Engineering, 3rd Year Examination, 2017

(2nd Semester)

INTRODUCTION TO STATISTICAL AND PROBABILISTIC METHODS

Time : 3 hrs. Full Marks : 100 (50 for each part)

Use a Separate Answer-script for Each Part

PART II

Answer Any Three Questions

Two Marks Reserved for Neat and Well Organized Answers

1. (a) If A and B are the least squares estimators of α and β respectively in a linear regression model, and if we assume that the random errors are independent normal random variables having mean 0, then establish that A and B are unbiased estimators.

(b) Discuss the statistical inferences concerning α and β. 08+08

2. (a) In a sample from a normal population having unknown mean and known variance, explain how an interval can be specified, for which we can have a certain degree of confidence, for the mean to lie within.

(b) What are maximum likelihood estimators ?

(c) An astronomer wants to measure the distance from her observatory to a distant star. However, due to atmospheric disturbances, any measurement will not yield the exact distance *d*. As a result, the astronomer has decided to make a series of measurements and then use their average value as an estimate of the actual distance. If the astronomer believes that the values of the successive measurements are independent random variables with a mean of *d* light years and a standard deviation of two light years, how many measurements need she take to be at least 95 percent certain that her estimate is accurate to within$\pm$ 0.5 light years.

06+04+06

3. (a) Establish the relationships between the mean and variance of sample and those of the population. State and discuss the Central Limit Theorem.

(b) An insurance company has 25,000 automobile policy holders. If the yearly claim of a policy holder is a random variable with mean 320 and standard deviation 540, approximate the probability that the total yearly claim exceeds 8.3 million.

08+08

4. (a) Justify the statement – 'If $X_1.....X_n$ is a sample from a normal population having mean μ and variance σ^2, then $Z = \sqrt{n}\,(\overline{X} - \mu)/\sigma$ will have a standard normal distribution'; Also show that $\sqrt{n}\,(\overline{X} - \mu)/S$ has a t-distribution with (n-1) degrees of freedom. All notations have usual significance.

(b) Two proof-readers were given the same manuscript to read. If proof-reader 1 found n_1 errors, and proof-reader 2 found n_2 errors, with $n_{1,2}$ of these errors being found by both proof-readers, estimate N, the total number of errors that are in the manuscript.

10+06

5. Write short notes on any two :

(a) The F-Distribution (b) Significance Levels (c) The Bayes Estimators 08+08

For solving the numerical problems the standard normal probability distribution function table provided below may be used with linear interpolation / extrapolation.

z	0.1	0.2	0.3	1.8	1.9	2.0	3.2	3.3	3.4
$\phi(z)$	0.5398	0.5793	0.6179	0.9641	0.9713	0.9772	0.9993	0.9995	0.9997