13. a) Expand f(x) = x, in Fourier Series on the interval $-\pi \le x \le \pi$.

b) Find a series of cosines of multiples of x which will represent f(x) = x on the closed interval $0 \le x \le \pi$. 6+4

14. a) State and prove the fundamental theorem of integral calculus.

b) Prove that $\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4 - x^2 + x^3}} < \frac{\pi}{6}$ 6+4

Bachelor of Engineering in Electrical Engineering Examination, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - IF

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

Symbols & Notations have their usual meanings.

Answer ***any five*** questions.

1. a) Examine the continuity of the function

$$f(x,y) = \begin{cases} \frac{x^4 + y^4}{x - y}, & \text{when } x \ne y \\ 0, & \text{when } x = y \end{cases}$$

at (0, 0).

b) Evaluate $\lim_{x \to 0} \frac{x^{1/2} \tan x}{(e^x - 1)^{3/2}}$ 5+5

2. a) State Rolle's theorem. Are the three conditions of Rolle's theorem necessary ? Justify your answer.

b) Use mean value theorem of appropriate order to prove that $\sin x > x - \frac{x^3}{3!}$

c) Give geometrical interpretation of mean value theorem. (Lagrange's form) 4+4+2

3. a) Prove that $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}} = 0$

b) Discuss the existence of extreme value of $f(x) = x^{1/3}$

c) Let f be a real valued function defined over (–1, 1) such

that $f(x) = x\cos\frac{1}{2}, \text{ for } x \neq 0$

$= 0, \quad \text{for } x = 0$

Does mean value theorem hold for f in (–1, 1) ? 4+2+4

4. a) State Euler's theorem for hemageneous function of three variables.

If $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x-y}\right)$, then find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}.$$

b) If $u = \left(1-2xy+y^2\right)^{-1/2}$, then show that

$$\frac{\partial}{\partial x}\left\{\left(1-x^2\right)\frac{\partial u}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{y^2\frac{\partial x}{\partial y}\right\} = 0$$

6+4

10. Test the convergence of the following :

i) $\displaystyle\int_0^{\infty} \frac{dx}{1+x^2}$

ii) $\displaystyle\int_0^{\pi/0} \frac{x^m}{\sin^n x}dx$

iii) $\displaystyle\int_0^{\infty} x\, e^{-x^2}dx$ 10

11. a) Show that $B(m,n) = 2\displaystyle\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta$

b) Prove that $\displaystyle\int_0^{\pi/2} \sqrt{\tan x}\, dn = \pi/\sqrt{2}$

c) Show that $\displaystyle\int_0^{\pi/2} \sin^p x\, dn \cdot \int_0^{\pi/2} \sin^{p+1} x\, dn = \frac{\pi}{2(p+1)}$ 10

12. a) Evaluate $\displaystyle\iint_R \sin(x+y)\,dx\,dy$ over the region $R:\left\{0 \le x \le \pi/2;\ 0 \le y \le \pi/2\right\}$

b) Evaluate $\iint\left(x^2+y^2\right)dx\,dy$ over the region enclosed by the triangle having its vertics at (0, 0), (1, 0), (1, 1).

5+5

[Turn over

PART - II

Answer ***any five*** questions.

8. a) Explain the concept of Riemann integrability of a bonded function f(x) on [a, b].

   b) Show that the contant function f(x) = K is intigrable.

   c) A function f is defined on [0, 1] by

   f(x) = 1, if x is rational

   = 0, if x is irrational

   Is f Riemann integrable on [0, 1] ? – Explain. 3+4+4

9. a) Show that the function f defined by

$$f(x)=\frac{1}{2^n},\ \text{when}\ \frac{1}{2^{n+1}}<n\le\frac{1}{2^n},\ n=0,1,2\ ....$$
$$=0,\ \text{when}\quad x=0$$

   is integrable on [0, 1] and hence evaluate $\int_0^1 f(x)dx$.

   b) If a function f(x) is continuous on [a, b], then show that there exists a number $\xi\in(a,b)$ such that

$$\int_a^b f(x)dx=(b-a)f(\xi)$$ 5+5

5. a) Use Lagrange's method of undetermined multiplier to obtain the extreme values of $x^2+y^2+z^2$ if it exist when $x+y+z=3a$

   b) If $u=f(xyz)$, then show that $xu_x=yu_y=zu_z$ 7+3

6. a) The roots of his equation in $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ are u, v, w then prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$$

   b) Suppose f be a function defined by

$$f(x,y)=\begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{when } x^2+y^2\ne 0\\ 0, & \text{when } x^2+y^2=0\end{cases}$$

   Show that $f_{xy}(0,0)\ne f_{yx}(0,0)$. 5+5

7. a) If $\phi''(x)\ge 0$, for all x in (a, b), then prove that

$$\phi\left[\frac{1}{2}(x_1+x_2)\right]\le\frac{1}{2}\left[\phi(x_1)+\phi(x_2)\right]$$

   where x_1, x_2 be any two points in the interval.

   b) If f is continuous on [a, b] and $f'(x)>0$ in (a, b), then prove that f is strictly increasing in [a, b].
   Also state the converse of this result is true or false. Explain with example. 5+5

[Turn over