BACHELOR OF CONSTRUCTION ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester)

Mathematics - II E

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Use the method of separation of variables to solve for u(x,t) where u(x,t) satisfies

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Hence solve the following boundary value problem for u(x,t).

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \ell, t > 0,$$

$u \to 0$ as $t \to \infty$.

$$\frac{\partial u}{\partial x} = 0 \text{ for } x = 0$$

$u = 0$ for $x = \ell$

$u = u_0$ for $t = 0$, where u_0 is a constant. 10+10

2. (a) Find the equation of sphere of which $x+y+z=3$ and $x+y+z=9$ are two tangent planes and whose centre lies on both the planes $2x-y=0$ and $3x-z=0$.

   (b) Find the equation of the cylinder whose guiding curve is the intersection of surface S : $x^2+y^2+z^2=4$ with plane $x+y+z=0$ and generators parallel to the line whose direction ratio is (2,–1,2).

   (c) A variable point P is such that its distance from y axis is always equal to its distance from the plane $x-z=1$. Show that locus of P is $x^2+z^2+2xz+2x-2z=1$.

   8+7+5

3. (a) Find the equation of cone whose vertex is (1,2,3) and the guiding curve is $\frac{x^2}{9}+\frac{y^2}{4}=1,\ z=0$.

   (b) P is a variable point such that the sum of its distances from the coordinate planes is always equal to its distance from origin. Find the locus of P.

   (c) Given that $\vec{\alpha}=\hat{i}+\hat{j}-2\vec{k},\ \vec{\beta}=-\hat{i}+2\hat{j}+3\vec{k},\ \vec{r}=5\hat{i}+8\vec{k}$.

   Find the scalars c and d such that $\vec{r}=c\vec{\alpha}-d\vec{\beta}$ is perpendicular to both $\vec{\alpha}$ and $\vec{\beta}$. 7+7+6

4. (a) Find the Fourier transform of

   (i) $f(x)=\begin{cases}1, & -1<x<1\\ 0, & \text{otherwise}\end{cases}$

   (c) The probability density function of a continuous random variable X is

   $$y = K(x-1)(2-x),\ 1 \le x \le 2.$$

   Determine

   (i) The value of constant K. (ii) The cumulative distribution function. (iii) The probability that X is less than $^5/_4$. (iv) The probability that X is greater than $^3/_2$. (v) The probability that X lies between $^5/_4$ and $^3/_2$.

   4+6+10

8. (a) What are sampling distribution and standard error?

   (b) A simple random sample of size 5 is drawn without replacement from a finite population consisting of 41 units. If the population standard deviation is 6.25, what is the standard error of sample mean? (Use finite population correction).

   (c) For a population of six units, the value of characteristics x are 3, 9, 6, 5, 7, 10. Consider all possible samples of size two from the above population and show that the mean of the sample means is exactly equal to population mean.

   (d) Show that $\text{var}(aX+b)=a^2\ \text{Var}(X)$, where X is a discrete random variable. 4+6+6+4

—— X ——

6. (a) Solve the following LPP using graphical method.

Minimize $z = 3x + 5y$

subject to $2x + 3y \geq 12$

$-x + y \leq 3$

$x \leq 4$

$y \geq 3,$

$x \geq 0, y \geq 0.$

(b) Define feasible and optimal solutions of an LPP. Show that the feasible region for the following problem is unbounded.

Maximize $z = 3x_1 + 2x_2$

subject to $x_1 - x_2 \leq 1$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

Does the LPP admits of any solution? 10+10

7. (a) Show that the mean of a Binomial distribution is np where n and p have their usual meaning.

(b) A random variable has the following probability distribution.

x :	4	5	6	8
p :	0.1	0.3	0.4	0.2

Find the expectation and the standard deviation of the random variable.

(ii) $f(x) = e^{-a|x|}, a > 0$

(iii) $f(x) = \begin{cases} 1-|x|, & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

(iv) $\frac{d^3 f}{dx^3}$ 2+8+5+5

5. (a) Define frequency distribution. How many types of frequency distributions are there? What is relative frequency?

(b) Obtain the median of the following frequency distribution.

x :	1	2	3	4	5	6	7	8	9
f :	8	10	11	16	20	25	15	9	6

(c) Define Mean deviation. Calculate the mean deviation for the following data :

Marks :	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	12	18	27	20	17	6

4+8+8