Ex./CON/MATH/T/112/2017(S)

BACHELOR OF CONSTRUCTION ENGG. EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - I E

Time : Three hours — Full Marks : 100

Answer any ***five*** questions.

1. (a) Show without expanding

(i)

$$\begin{vmatrix} a & b & c \\ b+c & c+a & a+b \\ a^2 & b^2 & c^2 \end{vmatrix} = -(a-b)(b-c)(c-a)(a+b+c)$$

(ii)

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

5+5

(b) Solve by Crammer's rule :

$x-3y+z=2,\ 3x+y+z=6,\ 5x+y+3z=3.$ 5

(Turn over)

(c) For three distinct numbers a, b, c show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$ can not be zero. 5

2. (a) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then by mathematical induction show that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer. 5

(b) Find the inverse of

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$ 5

(c) Find the rank and normal form of the matrix

$$\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$ 5

(d) Solve by matrix method : $x+y+z=3$, $x+2y+3z=4$ and $x+4y+9z=6$. 5

7. (a) Find the divergence and curl of the vector $\vec{A} = (xyz)\vec{i} + \left(3x^2y\right)\vec{j} + (xz^2 - y^2z)\vec{k}$ at the point (2,–1,1). 5

(b) If $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\text{Curl}(\vec{a}\times\vec{r}) = 2\vec{a}$. 5

(c) Prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$. Where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. 5

(d) Find the directional derivative of the function $f(x,y,z) = xy^2 + yz^3$ at the point (2,–1,1) in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$.. 5

8. (a) If $\vec{F} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, then evaluate the line integral $\int_C \vec{F}.\,d\vec{r}$ from (0,0,0) to (1,1,1) along the following path C : $x = t,\ y = t^2,\ z = t^3$.

(b) Verify divergence theorem for $\vec{F} = 2x^2y\vec{i} - y^2\vec{j} + 4xz^2\vec{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ and $x = 2$.

(c) Evaluate by Stoke's theorem $\oint_C (yzdx + zxdy + xydz)$ where C is the curve : $x^2 + y^2 = 1,\ z = y^2$.

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5. (a) Show that $\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x}\, dx = \pi$. 5

(b) Evaluate : $\int_0^{\infty} e^{-x^2} x^{\alpha}\, dx,\ \alpha > -1$. 5

(c) Find the value of $\int_0^1 \frac{dx}{(1-x^6)^{1/6}}$. 5

(d) Prove that $B(x,y) = \int_0^{\infty} \frac{t^{y-1}}{(1+t)^{x+y}}\, dt$, x, y > 0. 5

6. (a) Determine 'a' such that

$$\lim_{x\to 0} \frac{a\sin x - \sin 2x}{\tan^3 x}$$ exists and = 1. 4

(b) Evaluate : $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x}$. 4

(c) If $y = \sin(m\sin^{-1} x)$, then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2 - n^2)y_n = 0$$

where $y_n = \frac{d^n y}{dx^n}$. 7

(d) Find y_n, when $y = x^3 \log x$. 5

3. (a) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$ 7

(b) Prove that $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)} dx = \frac{\pi}{2}\log(1+a)$. 6

(c) State and prove Darbouse's theorem. 7

4. (a) If M, m be the least upper and greatest lower bounds of an integrable function f(x) on $a \le x \le b$ then show that $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$. 5

(b) Show that $\lim_{x\to 0} \int_0^x \frac{\sin t\, dt}{x^2} = \frac{1}{2}$. 5

(c) Examine the convergence of the following integraly :

(i) $\int_0^{\infty} \frac{\cos x\, dx}{1+x^2}$

(ii) $\int_0^{\infty} \frac{x^{3/2}}{b^2x^2 + c^2} dx$ 5+5