B.C.S.E 3rd Year 2nd Semester Examination 2017

DESIGN AND ANALYSIS OF ALGORITHMS

Time: **Three hours** Full Marks: **100**

Answer Question #1 and #2 and any three from the rest

1. a) Prove that the following iterative algorithm for the addition of natural numbers is correct:

```
add(y, z) {
    x = 0; c = 0; d = 1;
    while (y > 0) ∨ (z > 0) ∨ (c > 0) {
        a = y mod 2; b = z mod 2;
        if a ⊕ b ⊕ c then x = x + d;
        c = (a ∧ b) ∨ (b ∧ c) ∨ (c ∧ a); d = 2d; y = ⌊y/2⌋; z = ⌊z/2⌋;
    };
    return x;
}
```

b) Prove that the following recursive algorithm for multiplication of natural numbers is correct:

```
multiply(y, z) {
        if z = 0 then return 0
        else
            if z is odd the return (multiply(2y, ⌊z/2⌋ +y));
            else return (multiply(2y, ⌊z/2⌋));
}
```

10 + 10

2. a) Prove that (i) ⌈log n ⌉= O(n), (ii) n^2-3n-18= Ω(n) and (iii) n^3- 3n^2-n+1= Θ(n^3).

b) For the following algorithm provide the O, Ω and Θ analysis:

```
m(n){
    r = 0;
    for i = 1 to n − 1
      for j = i + 1 to n
          for k = 1 to j
              r = r + 1;
    return r;
}
```

10 + 10

3. a) Formulate a recursive algorithm for finding the largest of a set of numbers as a divide and conquer strategy and prove its correctness.
   c) Is insertion sort a stable sorting method?

   10+8+2

4. State the continuous knapsack problem and provide a greedy solution and show that it is correct.

   5 + 5 + 10

5. What is the principle of optimality? Does it always hold? Formulate the 0/1 knapsack problem as a dynamic programming problem. Formulate the matrix product chains problem as dynamic programming problem.

   4 + 4 + 6 + 6

6. a) Find the minimum time any comparison-based sorting algorithm can take. If there are four data then what is the minimum number of comparisons that would sort them and by which algorithm? b) Provide a schematic algorithm for sorting disk files.

   8 + 12

7. Write an algorithm for construction, insertion, deletion and element search in an AVL-tree.

   20
