BCSE Examination, 2017
(2nd Year, 2nd Semester)
MATHEMATICS
Paper- V

TIME: 3 Hours. **FULL MARKS: 100**

Attempt any five questions. Each question carries 20 marks

1. a) One card is selected at random from 100 cards numbered 00, 01,...., 99. Find the probability that the sum of the digits on the selected card is 1, if it is known that their product is 0.

   b) The numbers 1, 2,...., n are arranged in random order. What is the probability that the numbers 1 and 2 are always together. 10 + 10

2. a) Prove that if A, B, C be mutually independent, then A and $B + C$ are independent event. Define distribution function of a random variable.

   b) What is the probability that in a collection of 500 people, only one person will have his birthday on the New Year's day?(Assume that a year has 365 days)

   10 + 10

3. a) A point P is taken at random on a line segment AB of length $2a$. Find the probability that the area of the rectangle $AP.PB$ will exceed $\frac{1}{2}a^2$.

   b) If X is Poisson distributed with parameter μ, then prove that

$$P(X \leq n) = \frac{1}{n!}\int_{\mu}^{\infty} e^{-x}x^n dx$$

   Where n is any positive integer. 10 + 10

4. a) State and prove the Central limit theorem.

   b) The probability density function of a random variable X is $2xe^{-x^2}$ $for\ x > 0$ and zero otherwise. Find the same for X^2.

   12 + 8

5. a) Show that in 2,000 throws with a coin, the probability that the number of heads lies between 900 and 1100 is at least 19/20.

   b) Define the Covariance of two random variables.

   Two random variables X, Y have the following joint probability density function:

$$f(x,y) = \begin{cases} k(4 - x - y) \, ; 0 \le x \le 2 \, ; 0 \le y \le 2 \\ 0, \; otherwise \end{cases}$$

Find the constant $k, Var(X) and\ Covariance(X, Y)$. 10 + 10

6. a) Obtain the steady state solution of $(M|M|1)$: $(\infty|FCFS)$ system and also find the expected value of the queue length n.

   b) State and prove the Law of large number. 12+8

7. a) The Pascal distribution is defined by $x_i = i, (i = 0, 1, 2 \ldots)$ and

$$f_i = \frac{1}{(1+\mu)}\left(\frac{\mu}{1+\mu}\right)^i, \; \mu > 0.$$

   Find the mean and variance of this distribution.

   b) State and prove the Tchebycheff's inequality. 10+10