Ref. No.: **Ex/MATH/CSE/T/211/2017(S)**

B.CSE. 2ND YR. 1ST SEMS SUPPLEMENTARY EXAM, 2017

Mathematics

(Paper-IV)

Full Marks:100 Time: Three Hours

Answer Question number 1. and any six from the rest.

1. Determine the radius of convergence and interval of convergence of the power series (4)

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$$

2. Find a power series solution of the initial value problem (16)

$$(x^3-1)\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+xy=0$$

Write atleast first three nonzero terms in each part of the series.

3. Use the method of Frobenious to find solution near $x = 0$ of the differential equation (16)

$$x^2\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}-2y=0$$

Write atleast first three nonzero terms in each part of the series.

4. (a) Prove that (10)

$$P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n,$$

where $P_n(x)$ is the Legendra polynomial of degree n. Hence find the expressions for P_0, P_1, P_2 and P_3.

(b) Write generating function of Legendre ploynomials. Use that function to prove (6)

i. $P_n(-1) = (-1)^n$

ii. $P_{2n+1}(0) = 0$

5. State the orthogonality property of Chebyshev ploynomials of first kind. Plot the graph of first five Tchebyshev polynomials of first kind. Find the Tchebyshev series expansion of $sin(cos^{-1}x)$. Write first five terms of the series. (16)

6. (a) Use the method of variation of parameters to find general solution of the equation (10)

$$\frac{d^2y}{dx^2} + 4y = sec^2 2x$$

(b) Solve (6)

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ ln\ x$$

7. (a) Find the residue of (6)

$$\frac{z(z-2)}{(z+4)^2(z-1)^2} \quad \text{at} \quad z = 1 \text{ and } z = 4.$$

(b) Find the Laurent series expansion of $f(z) = \frac{1}{1-z}$ in terms of the negative powers of z which will be valid if $|z| > 1$. Write atleast first four terms of the series. (5)

(c) Evaluate (5)

$$\oint_C z dz, \quad \text{where } C \text{ is the contour } |z| = 1$$

8. (a) Evaluate (8)

$\oint_C z dz$, where C is the line from $1+i$ to $3+i$ and then from $3+i$ to $3+3i$.

(b) Find the harmonic conjugates of (8)

$(i) f(x,y) = e^x \sin y$ $\quad\quad$ $(ii) g(x,y) = xy - x + y$

9. Find the Fourier series of the functions (16)

$$(a) \quad f(x) = \begin{cases} 0, & -\pi \leq x < 0 \\ 1, & 0 \leq x \leq \pi \end{cases}$$

$$(b) \quad f(x) = \pi - x, \quad -\pi \leq x \leq \pi$$