

B. CSE 2nd Year 1st Semester Supplementary Examination 2017

NUMERICAL METHODS

Time : 3 hours.

Full Marks : 100

**Answer question no.1 and any 4 from the rest.
All parts of same question should be answered together.**

1. a) What will be the maximum absolute error when an approximate number p is divided by another approximate number q . 3
- b) Derive Aitken's acceleration formula. 3
- c) Is it possible to get a solution of the following system of equations by Gaussian elimination method? 3

$$\begin{aligned}x - 2y + 3z &= 15 \\3x + 2y + 9z &= 18 \\x + y + 3z &= 10\end{aligned}$$

- d) Can Gauss- Seidel method be used for solving the following system of equations? Why? 2

$$\begin{aligned}9x + 2y + 3z &= 2 \\3x + y + z &= 5 \\x - 6y + 2z &= 10\end{aligned}$$

- e) Derive the recursive formula to find $1/N$ using Newton- Raphson method. 3
- f) Define Δ , ∇ and E . Prove that $E\nabla = \Delta$. 3
- g) Given the function $y = 1/x$, show that

$$y [x_0, x_1, \dots, x_n] = (-1)^n / (x_0 x_1 \dots x_n) \quad 3$$

2. a) Discuss Secant method for solution of non-linear equations. 6
- b) Derive the order of convergence of the same method. 8
- c) What is its advantage over Newton- Raphson method? 2
- d) Derive the recursive formula for evaluating $N^{1/3}$ by Chebyshev method. 4

3. a) Discuss Gauss- Jordan elimination method for matrix inversion. 6
 b) Calculate the number of multiplications / divisions required in the matrix inversion process. 6
 c) Find the inverse of the following matrix. 8

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

4. a) Define the terms eigenvalue and eigenvector. 2
 b) Find all the eigenpairs (λ_i, X_i) of the following matrix. 8

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

- c) Fit the function $y = A e^{Bx}$ to the following table of values by method of Least Squares. 10

x	0	1	2	3	4
y	1.5	2.5	3.5	5.0	7.5

5. a) Discuss Euler's method for solution of ordinary differential equations. 6
 b) Derive the condition for convergence of this method and also the expression for truncation error associated with this method. 4
 c) Solve the following initial value problem using Euler's method. 10

$$\frac{dy}{dx} = (x - y) / 2 \text{ with } y(0) = 1$$

Solution is required over $[0, 1]$ with $h = 1/8$.

Calculate the percentage error with the exact solution

$$Y = 3 e^{-x/2} + x - 2$$

6. a) Discuss Jacobi's iterative method for solution of linear simultaneous equations 6
 b) Write down the algorithm in matrix notation. 4
 c) Find out the condition of convergence of the method. 4
 d) Solve the following system of equations by Jacobi's iterative method. 6
 Solution is required corrected upto 4 decimal places.

$$\begin{aligned} 10x + 2y + z &= 9 \\ x + 10y - z &= -22 \\ -2x + 3y + 10z &= 22 \end{aligned}$$

7. a) Derive numerical integration formula using Simpson's $\frac{1}{3}$ rule. 10
b) Derive the expression for total truncation error associated with this method. 10
8. a) Discuss Lin's method for finding the complex roots of a polynomial equation. 10
b) Evaluate the following integral corrected upto 3 decimal places using Trapezoidal method.

$$\int_0^2 \{ 1 / (x^2 + 4) \} dx \quad 10$$
