B.CSE, Examination 2017
(2nd year, 1st Semester (Supplementary))
MATHEMATICS
Paper – VD(Old)

Full Marks: 100 Time: Three Hours

(Answer any five questions)

(Symbols/Notations have their usual meanings)

1) Solve the following differential equations

a) $(2D^3 - 3D^2 + 1)y(x) = 1 + e^x$ where $D = \frac{d}{dx}$ 5

b) $(D^3 - 3D^2 + 3D - 1))y(x) = e^x(x+1)$ 5

c) $(D^2 - 3D + 2)y(x) = \frac{e^x}{1+e^x}$ 5

d) $(D^2 - 5D + 6)y(x) = x^3 e^{2x}$ 5

2a) Solve the following differential equation by the method of variation of parameters

$$(D^2 + 1)y(x) = Cosecx$$ 8

b) Solve the following differential equation

$$(x^2D^2 - xD - 3)y(x) = x^2 \, logx$$ 8

c) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$, in terms of Legendre polynomials. 4

3a) Solve $\frac{d^2y}{dx^2}$+(x-1)$\frac{dy}{dx} + y = 0$, in powers of $x = 2$ 10

b) Show that for a Legendre function $P_n(x)$

$$(1 - 2xh + h^2)^{-\frac{1}{2}} = \sum_{n=0}^{\infty} h^n P_n(x)$$ 10

4a) Prove that $x\, P_n^{/}(x) = P_{n-1}^{/}(x) + nP_n(x)\; and P_n(1) = 1$ 10

b) Prove that

i) $\frac{\sqrt{1-x^2}}{1-2xt+t^2} = \sum_0^{\infty} U_{n+1}(x)t^n$

ii) $(1 - x^2)\, T_n^{/}(x) = \frac{n}{2}[T_{n-1}(x) - T_{n+1}(x)]$ 10

5 a) State Dirichlet's condition for a Fourier series expansion of function. Find the Fourier

series for the function

$$f(x) = -\frac{\pi}{4} \ for -\pi \leq x \leq 0$$
$$= \frac{\pi}{4} \ for \ 0 \leq x \leq \pi$$ 3+7

b) Obtain the half range Cosine series for

$$f(x) = Kx \ for \ 0 \leq x \leq \frac{l}{2}$$
$$= k(l-x) \ for \ \frac{l}{2} \leq x \leq l$$ 10

6a) A periodic function of period 4 is defined as

$f(x) = |x|; \quad -2 \leq x \leq 2.$

Find its Fourier expansion. 10

b) A periodic function of period 2π is defined as

$$f(x) = x + \pi \quad for \ 0 \leq x \leq \pi$$
$$= -x - \pi \ for \ -\pi \leq x \leq 0$$

Find its Fourier series. 10

7a) Examine whether the function

$$f(z) = \frac{x^2y^3(x+iy)}{x^4+y^6}, for \ z \neq 0$$
$$= 0, \qquad for \ z = 0$$

is analytic or not at origin. 8

b) If $f(z) = u(x,y) + v(x,y)$ is an analytic function of $z = x + iy$, show that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f'(z)|^2 = 4\,|f'(z)|^2$$ 8

c) Let $f(z) = u(x,y) + i\,v(x,y)$ be an analytic function.
If $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ then find $v(x,y)$ also express $f(z)$ in terms of z. 4

8a) Expand the $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurant series valid for $i)\ |z| > 3, ii)\ 0 < |z+1| < 2,$ 6

b) Determine the poles and the residue at each point of the function

$$f(z) = \frac{z^2}{(z-1)^2\,(z+2)}$$ 7

c) Evaluate $\int_0^{2+i} \bar{z}^2\, dz$, (a) along the line $y = \frac{x}{2}$ and (b) along the real axis to 2 and then vertically to $2 + i$. 7

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