(b) Prove that the asymptotes of the cubic

$(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ 10

form a triangle of area a^2.

—— X ——

Ex./CSE/MATH/T/119/2017(OLD)(S)

BACHELOR OF COMPUTER SC. ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - II D (OLD)

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

1. (a) Define open set, interior point and limit point of a set in real number $\mathbb{R}$. 6

   (b) Prove that every convergent sequence is bounded. 10

   (c) Show that the series $1+\frac{1}{2!}+\frac{1}{3!}+\ldots$ is convergent. 4

2. (a) State Cauchy's general principle of convergence of an infinite series. 10

   Prove that if $u_n > 0$ and $\underset{n\to 0}{\mathrm{Lt}}(u_n)^{\frac{1}{n}} = \rho$ then

   (i) $\sum u_n$ convergese if $\rho < 1$

   (ii) diverges if $\rho > 1$

(b) Test the convergence of the series 5x2=10

(i) $\frac{5}{1.2.4}+\frac{7}{2.3.5}+\frac{9}{3.4.6}+....$

(ii) $\sum_{n=2}^{\infty}\frac{\log n}{\sqrt{n+1}}$

3. (a) State Rolle's theorem and give its geometrical interpretation. Are the conditions of the Rolle's theorem necessary ? Verify Rolle's theorem for

$$f(x) = 2x^3 + x^2 - 4x + 289$$

in some suitable closed interval. 10

(b) Find the value of y_n for $x=0$ when $y=e^{a\sin^{-1}x}$. 10

4. (a) Define homogeneous function of degree n in two variables.

If f(x,y) be a homogeneous function of x and y of degree n then prove that

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=n\,f(x,y)$$ 10

(b) If $u = \log(x^2+y^2+z^2)$ prove that

$$x\frac{\partial^2 u}{\partial y\partial z}=y\frac{\partial^2 u}{\partial z\partial x}=z\frac{\partial^2 u}{\partial x\partial y}$$ 5

(c) If $u = e^{xyz}$, prove that

$$\frac{\partial^3 u}{\partial x\partial y\partial z}=\left(1+3xyz+x^2y^2z^2\right)e^{xyz}$$ 5

5. (a) State and prove Leibnitz's theorem of nth derivative of the product of two functions. 10

(b) Evaluate the following limits :

(i) $\underset{n\to 0}{\mathrm{Lt}}(\cos x)^{\frac{1}{x^2}}$

(ii) $\underset{n\to 1}{\mathrm{Lt}}\left\{\frac{x}{x-1}-\frac{1}{\log x}\right\}$ 10

6. (a) State and prove fundamental theorem of integral calculus. 10

(b) Compute the value of the integral $\int_0^1 x^2dx$ by Riemann integral theory. 10

7. (a) Show that

$$\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2n+1)}{2^{2n}\,\Gamma(n+1)}$$ 10

(Turn over)