Bachelor of Computer Science & Engineering Examination 2017 Special (OLD)

(First Year, First Semester)

MATHEMATICS - ID

Time : Three Hours | Full Marks : 100

The figures in the margin indicate full marks

Answer Q. No. 9 and any **six** questions from Q. Nos. 1 – 8.

1. Let A, B, C be three subsets of a set X.

   (a) Show that $A\Delta(B\Delta C) = (A\Delta B)\Delta C$. 8

   (b) If $A\Delta C = B\Delta C$, then prove that $A = B$. 8

2. (a) Define *reflexive* and *symmetric relations* on a nonempty set. Let S be a set with 40 elements. Find the number of reflexive and symmetric relations that can be defined on S. 8

   (b) What is an *equivalence relation* on a nonempty set? Find all equivalence relations on $\{a, b, c\}$. 8

3. (a) Define a *surjective* function. Let $f : A \longrightarrow B$ and $g, h : B \longrightarrow C$ be functions, where A, B, C be three nonempty sets. If $g \circ f = h \circ f$ and f is surjective, then prove that $g = h$. 8

   (b) Let X and Y be two nonempty sets and $f : X \longrightarrow Y$ be a function. Prove that f in one-to-one if and only if for all subsets A, B of X, $f(A \cap B) = f(A) \cap f(B)$. 8

4. (a) If n is a positive integer greater than 1, then show that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not an integer. 8

   (b) Prove that $5^n + 3$ is divisible by 4 for all natural numbers n. 8

5. (a) Define a *permutation* and an *odd permutation*. Find the number of odd permutations in S_6. 8

   (b) Let $\alpha = (2\ 4\ 7\ 9\ 5)(8\ 1\ 3)$, $\beta = (5\ 6\ 7\ 1)(2\ 8\ 6)$ and $\gamma = (4\ 8\ 9\ 7\ 6\ 5)(3\ 2\ 1)$ in S_9. Express $\alpha^2\beta^{-3}\gamma^{-2}$ as a product of disjoint cycles. 8

6. (a) Define a *countable* set. Prove that the set $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is countable, where $\mathbb{Q}$ is the set of all rational numbers. 8

   (b) Prove that set of all irrational numbers is uncountable. 8

7. (a) What is a *cardinal number* $|A|$ of a set A? Prove that the set of all real functions defined on the closed unit interval has the cardinal number 2^c, where c is the cardinal number of the set $\mathbb{R}$ of all real numbers. 8

   (b) Prove that $|A| < |\mathscr{P}(A)|$ for any set A, where $\mathscr{P}(A)$ is the power set of A. 8

8. (a) Define a *well-ordered set.* Show that the set of all natural numbers is well-ordered. 8

   (b) Let $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x < 1 \text{ and } 0 \leqslant y < 1\}$ and $B = \{x \in \mathbb{R} \mid 0 \leqslant x < 1\}$. Show that $|A| = |B|$. 8

9. Prove that every infinite set A has a proper subset B such that $|A| = |B|$. 4