Ex./CSE/MATH/T/113/2017(OLD)(S)

# BACHELOR OF COMPUTER SC. ENGINEERING EXAMINATION, 2017

## (1st Year, 1st Semester, Supplementary)

## Mathematics - I D (OLD)

Time : Three hours | Full Marks : 100

*The figures in the margin indicate full marks*

Answer Q. No. 9 and any six questions from Q. Nos. 1 – 8.

1. Let $A, B, C$ be three subsets of a set $X$.

   (a) Show that $A\Delta(B\Delta C) = (A\Delta B)\Delta C$. 8

   (b) If $A\Delta C = B\Delta C$, then prove that $A = B$. 8

2. (a) Define *reflexive* and *symmetric relations* on a nonempty set. Let $S$ be a set with 40 elements. Find the number of reflexive and symmetric relations that can be defined on $S$. 8

   (b) What is an *equivalence relation* on a nonempty set? Find all equivalence relations on $\{a, b, c\}$. 8

3. (a) Define a *surjective* function. Let $f : A \longrightarrow B$ and $g, h : B \longrightarrow C$ be functions, where $A, B, C$ be three nonempty sets. If $g \circ f = h \circ f$ and $f$ is surjective, then prove that $g = h$. 8

   (b) Let $X$ and $Y$ be two nonempty sets and $f : X \longrightarrow Y$ be a function. Prove that $f$ in one-to-one if and only if for all subsets $A, B$ of $X$, $f(A \cap B) = f(A) \cap f(B)$. 8

4. (a) If $n$ is a positive integer greater than 1, then show that $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not an integer. 8

   (b) Prove that $5^n + 3$ is divisible by 4 for all natural numbers $n$. 8

5. (a) Define a *permutation* and an *odd permutation*. Find the number of odd permutations in $S_6$. 8

   (b) Let $\alpha = (2\ 4\ 7\ 9\ 5)(8\ 1\ 3)$, $\beta = (5\ 6\ 7\ 1)(2\ 8\ 6)$ and $\gamma = (4\ 8\ 9\ 7\ 6\ 5)(3\ 2\ 1)$ in $S_9$. Express $\alpha^2\beta^{-3}\gamma^{-2}$ as a product of disjoint cycles. 8

6. (a) Define a *countable* set. Prove that the set $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is countable, where $\mathbb{Q}$ is the set of all rational numbers. 8

(b) Prove that set of all irrational numbers is uncountable. 8

7. (a) What is a *cardinal number* $|A|$ of a set $A$? Prove that the set of all real functions defined on the closed unit interval has the cardinal number $2^c$, where $c$ is the cardinal number of the set $\mathbb{R}$ of all real numbers. 8

(b) Prove that $|A| < |\mathscr{P}(A)|$ for any set $A$, where $\mathscr{P}(A)$ is the power set of $A$. 8

8. (a) Define a *well-ordered set.* Show that the set of all natural numbers is well-ordered. 8

(b) Let $A = \{(x, y) \in \mathbb{R}^2 \mid 0 \leqslant x < 1 \text{ and } 0 \leqslant y < 1\}$ and $B = \{x \in \mathbb{R} \mid 0 \leqslant x < 1\}$. Show that $|A| = |B|$. 8

9. Prove that every infinite set $A$ has a proper subset $B$ such that $|A| = |B|$. 4