Bachelor of Computer Science & Engineering Supplementary Examination 2017

(First Year, First Semester)

MATHEMATICS - I

Time : Three Hours   Full Marks : 100

The figures in the margin indicate full marks

Answer Q. No. 9 and any **six** questions from Q. Nos. 1 – 8.

1. (a) Let A, B, C be three subsets of a set X. If $A\Delta C = B\Delta C$, then prove that $A = B$. 8

   (b) Define *reflexive* and *anti-symmetric relations* on a nonempty set. Let S be a set with 40 elements. Find the number of reflexive and anti-symmetric relations that can be defined on S. 8

2. (a) Define a *permutation* and an *even permutation.* Find the number of even permutations in S_{15}. 8

   (b) Let $\alpha = (2\ 5\ 7\ 9\ 4)(3\ 1\ 8)$, $\beta = (5\ 1\ 7\ 6)(2\ 8\ 6)$ and $\gamma = (4\ 6\ 9\ 5\ 8\ 7)(3\ 2\ 1)$ in S_9. Express $\alpha^2\beta^{-3}\gamma^{-2}$ as a product of disjoint cycles. 8

3. (a) Define an *uncountable* set. Prove that the set of irrational numbers is uncountable. 8

   (b) Prove that $|A| < |\mathscr{P}(A)|$ for any set A, where $\mathscr{P}(A)$ is the power set of A. 8

4. (a) Find the truth table of $[(p \to q) \vee (q \to r)] \wedge (p \to r)$. 8

   (b) Let $A = \{1, 2, \ldots, 10\}$. Consider each of the following sentences. If it is a statement, then determine its truth value. If it a proposional function, determine its truth set.

      (i) $(\forall x \in A)(\exists y \in A)(x + y < 4)$. 4

      (ii) $(\forall x \in A)(\forall y \in A)(x + y < 4)$. 4

5. (a) If α, β, γ are the direction cosines of a line, then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$. 8

   (b) If $\vec{\alpha} = 2\vec{i} - 12\vec{j} + 3\vec{k}$, $\vec{\beta} = 4\vec{i} + \vec{j} + 7\vec{k}$ and $\vec{\gamma} = 3\vec{i} + 4\vec{j} + 3\vec{k}$. Find the vector $\vec{\alpha} \cdot (\vec{\beta} \times \vec{\gamma})$. 8

6. (a) Find the equation of the plane which passes through the point $(2, -1, 1)$ and is orthogonal to each of the planes $x - y + z = 1$ and $3x + 4y - 2z = 0$. 8

(b) Show that the angle between the diagonals of a cube is $\cos^{-1}\left(\frac{-1}{3}\right)$. 8

7. (a) Show that the spheres $x^2+y^2+z^2=25$ and $x^2+y^2+z^2-18x-24y-40z+225=0$ touch each other. Find there point of contact. 8

(b) Find the equation of the right circular cone whose vertex is at the origin, axis is $\frac{x}{3}=\frac{y}{2}=\frac{z}{4}$ and semivertical angle is 45°. 8

8. (a) If $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are three unit vectors satisfying $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\vec{0}$, where $\vec{0}$ is the zero-vector, then find $\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}\cdot\vec{\gamma}+\vec{\gamma}\cdot\vec{\alpha}$. 8

(b) A point moves in such a way that the sum of the squares of its distances from the six faces of a cube is constant. Show that its locus is a sphere. 8

9. Let $S=\{n\in\mathbb{N} \mid n \text{ divides } 144\}$. Define a *lattice*. Show that $(S,\vee,\wedge)$ is a lattice, where $a\wedge b=\gcd\{a,b\}$ and $a\vee b=\text{lcm}\ \{a,b\}$. 4