Ex./CSE/MATH/T/125/2017(OLD)

BACHELOR OF COMPUTER SC. ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - IV D

Time : Three hours | Full Marks : 100

Answer any ***five*** questions.

1. (a) Prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$ 6

(b) Explain with reasons if $(A+B)(A-B) = A^2-B^2$ hold true for square matrices. 4

(c) Find the inverse of the matrix $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and show that $A^3 = A^{-1}$. 7

(d) If possible solve the system of equations

$x-2y+z-\omega=-1$

$3x-2z+3\omega=-4$

$5x-4y+\omega=-3$ 3

(Turn over)

2. (a) Find the rank of the matrix $\begin{pmatrix} 2 & 0 & 2 & 2 \\ 3 & 4 & -1 & -9 \\ 1 & 2 & 3 & 7 \\ -3 & 1 & -2 & 0 \end{pmatrix}$ and reduce to its normal form. 7

(b) Solve

$x + y + z + \omega = 0$

$x + 3y + 2z + 4\omega = 0$

$2x + z - \omega = 0$ 6

(c) Define a subspace of a vector space. Examine whether W is a subspace of V where

(i) $V = R^3$, $W = \{(x,y,z) : x - 3y + 2z = 0\}$

(ii) V = the space of all square matrices of order 3

W = the set of all non-singular matrices of order 3. 2+3+2

3. (a) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$. 9

(b) Prove that the eigen vectors corresponding to distinct eigen values are linearly independent. 5

(b) Let p be a prime. Prove that (Zp, +, *) is a field. 6

(c) Show that a polynomial of degree n over a field F can have at most n roots in any extension field. 6

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(c) Prove that a set of n non-zero vectors $\{\alpha_1, \alpha_2, ..., \alpha_n\}$ is linearly dependent iff at least one of α_i is a linear combination of preceeding vectors. 6

4. (a) Show that (1,–1,3,2) , (1,1,0,3) , (0,1,0,0) and (0,0,1,0) forms a basis of R^4 and express (1,–3,4,7) as a linear combination. 7

(b) Find the basis and dimension of the subspace W spanned by (1,2,2), (3,2,1), (11,10,7) and (7,6,4). 5

(c) Define a basis of a vector space. Show that any two bases in a finite dimensional vector space have exactly same number of vectors. 8

5. (a) Prove that every finite dimensional vector space has a basis. 6

(b) If α, β, γ are linearly independent then show that $\alpha + \beta$, $\alpha - \beta$, $\alpha - 2\beta + \gamma$ are also so. 5

(c) Find a basis and dimension of the space of all skew symmetric matrices of order 3. 3

(d) Extend the following set (1,0,1,1), (–1,–1,0,0), (0,1,1,0) to a basis of R^4. 6

(Turn over)

6. (a) Show that H is a subgroup of a group (G, **.**) iff a, b $\in$ H implies $a^{-1}b \in H \ \forall$ a, b $\in$ H. 5

(b) Prove that intersection of a family of subgroups is a subgroup of a group but the union of two subgroups may not be so. 5

(c) Let H be a subgroup of a finite group G. Prove that

(i) $|aH| = |H| = |Ha| \ \forall a \in G$

(ii) O(H) divides O(G). 10

7. (a) If H, K are two subgroups of a group G, show that HK is a subgroup of G iff HK = KH. 7

(b) Define a normal subgroup of a group G. 2

(c) Define an ideal, a prime ideal and a principal ideal. Consider the ring of integers. For two positive integers a, b show that

(a) + (b) = (d) , d = gcd (a,b)

(a) . (b) = (c) , c = lcm (a,b)

where (x) is the principal ideal generated by x. 3+8

8. (a) Show that in a commutative ring R with identify, a proper ideal P of R is prime iff. R/P is an integral domain. 8