Ex./CSE/MATH/T/114A/2017(S)

BACHELOR OF COMPUTER SC. ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - II

Time : Three hours Full Marks : 100

Answer any ***five*** questions.

All symbols and notations have their usual meanings.

1. (a) Define bounded sequence.

If $x_n = \frac{3n-1}{n+1}$, prove that $\{x_n\}$ is monotonically increasing and bounded. 10

(b) Prove that the sequence $\{x_n\}$ where

$x_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n}$ is a convergent sequence. Estimate its limit. 10

2. (a) State Cauchy's general principle of convergence of an infinite series. Prove that if $u_n > 0$ and if

$\lim_{n\to\infty} (u_n)^{\frac{1}{n}} = \rho$, then

(i) $\sum u_n$ converges if $\rho < 1$.

(ii) diverges if $\rho > 1$.

(Turn over)

(b) Find the radius of convergence of the power series

$$x+\frac{(2!)^2}{4!}x^2+\frac{(3!)^2}{6!}x^3+\ldots.+\frac{(n!)^2}{(2n)!}x^n+\ldots. \qquad 10$$

3. (a) Show that the function

$$f(x,y)=\begin{cases} xy.\dfrac{x^2-y^2}{x^2+y^2}, & x^2+y^2\neq 0 \\ 0, & x=y=0 \end{cases}$$

is continuous at (0,0). 10

(b) If $v = 2\,\mathrm{Cos}^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then show that

$$x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}+\cot\frac{v}{2}=0. \qquad 10$$

4. (a) State and prove Lagrange's Mean Value theorem. 10

(b) Find the value of yn for $x = 0$ when $y = e^{a\sin^{-1}x}$. 10

5. (a) Prove that a bounded function f(x), having a finite number of points of discontinuity on [a,b] is integrable on [a,b]. 10

(b) Prove that the function f(x) defined by

$$f(x)=\begin{cases} x, & \text{when } x \text{ is rational} \\ -x, & \text{when } x \text{ is irrational} \end{cases}$$

is not integrable over [a,b], but |f| is integrable. 10

6. (a) Evaluate

(i) $\lim\limits_{x\to 0}\left(\dfrac{2^x+3^x}{2}\right)^{1/x}$

(ii) $\lim\limits_{x\to 0}\dfrac{x-\tan x}{x^3}$ 10

(b) Prove that $B(m,n)=\dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. 10

7. (a) Find the common area of the cardioide

$r = a(1+\mathrm{Cos}\,\theta)$ and the circle $r=\dfrac{3}{2}a$. 10

(b) Find the asymptotes of

$$x(x-y)^2-3(x^2-y^2)+8y=0. \qquad 10$$

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