Ex./CE/MATH/T/211/2017(OLD)(S)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2017

(2nd Year, 1st Semester, Supplementary)

Mathematics - III C (OLD)

Time : Three hours | Full Marks : 100

Answer any ***five*** questions.

1. (a) Find the Laplace transforms of the

   (i) $e^{-t}(2\,\text{Cos}\,5t - 3\,\text{Sin}\,5t)$

   (ii) $t^2\,(\text{Cos}\,3t)$

   (iii) $\int_0^t t^2 e^t\,dt$ 6

   (b) If f(t) be a periodic function with period $T > 0$ then show that

   $$L[f(t)] = \frac{\int_0^T e^{-st} f(t)dt}{1 - e^{-ST}}.$$ 4

   (c) Find the Inverse Laplace transform of

   (i) $F(S) = \dfrac{3s - 8}{S^2 - 4s + 20}$

   (ii) $F(S) = \dfrac{1}{(S-1)(S^2+4)}$ 5+5

(Turn over)

2. By using Laplace Transform method, solve the following differential equations (any ***two***) : 2x10

(a) $\frac{d^3y}{dt^2} - \frac{dy}{dt} = 2\,\text{Cos}\,t$, subject to the conditions :

$y(0) = 3, \left(\frac{dy}{dt}\right)_{t=0} = 2,$

$\left(\frac{d^2y}{dt^2}\right)_{t=0} = 1$

(b) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} = e^{-t}$, subject to the conditions :

$y(t) = 0, \frac{dy}{dt} = 1$ at $t = 0$.

(c) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = \text{Sin}\,t$, subject to the conditions

$y(0) = 1, \left(\frac{dy}{dt}\right)_{t=0} = 1.$

7. (a) Define regular and irregular singular points of a homogeneous second order linear differential equation. Find the series solution about the point x = 0 of the differential equation

$$9x(1-x)y'' - 12y' + 4y = 0$$

(b) Find series solution about the point x=0 of the differential equation

$$(1 + x^2)y'' + xy' - y = 0$$ 12+8

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5. (a) Solve the following differential equations :

(i) $\frac{dy}{dx} + x\sin 2y = x^2\cos^2 y$

(ii) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$

(iii) $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$

(b) Solve the following differential equation by the method of variation of parameters :

$$y'' - 2y' + y = e^x \log x$$ 12+8

6. (a) Solve the following differential equations :

(i) $\frac{d^2y}{dx^2} - 4y = x^2$

(ii) $(D^2 + 5D + 6)y = e^{-2x}\sin 2x$

(iii) $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin(2\log(1+x))$

(b) The differential equation for a simple pendulum is

$$\frac{d^2x}{dt^2} + \omega_0^2 x = F_0 \sin nt$$

where ω_0 and F_0 are constants. If initially $\frac{dx}{dt} = 0$, determine the motion when $\omega_0 = n$. 12+8

3. (a) Find the fourier series of f(x) :

$$f(x) = -\frac{1}{2},\ -\pi < x < 0$$

$$= \frac{1}{2},\ 0 < x < \pi$$

and $f(x + 2\pi) = f(x)$ 8

(b) Find the fourier series of f(x) where

$f(x) = 0,\ -\pi < x < 0$

$= 1,\ 0 < x < \pi$

Hence show that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} - \ldots.. + \ldots. = \frac{\pi}{4}$$ 12

4. (a) Prove the recurrence relations :

(i) $(2n+1)x\,P_n(x) = (n+1)p_{n+1}(x) + np_{n-1}(x)$

(ii) $np_n(x) = x\,p_n'(x) - p_{n-1}'(x)$

where $P_n(x)$ is Legendre polynomial. 10

(b) Prove the recurrence relations for Bessel functions :

(i) $x\,J_n'(x) = n\,J_n(x) - x\,J_{n+1}(x)$

(ii) $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$ 10