Ex./CE/MATH/T/112/2017(S)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary)

Mathematics - I C

Time : Three hours Full Marks : 100

Answer ***q.no. 9*** and any ***six*** from the rest.

Symbols and notations have their usual meaning.

1. (a) State Leibnitz theorem on higher order derivatives. Using the theorem find the nth derivative of

   (i) $y = x^2 \cos x$

   (ii) $y = x^3 e^{2x}$ 8

   (b) If $y = (x^2-1)^n$, prove that

   $(x^2-1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0.$ 8

2. (a) State and prove Lagrange's Mean Value theorem. 8

   (b) Use Mean Value theorem to show that

   $3+\frac{1}{28} < \sqrt[3]{28} < 3 + \frac{1}{27}.$ 5

   (c) Find the value of ξ in the Mean Value theorem for

   $f(x) = x(x-1)(x-2)$ in $[a, b]$

   where $a=0,\ b=\frac{1}{2}$. 3

(Turn over)

3. (a) Write down Maclaurin's series for log(1+x) with remainder after four terms. 8

(b) State L'Hospital's rule and using the rule find the limits of

(i) $\lim_{x\to 0} \frac{\log(1-x^2)}{\log \cos x}$

(ii) $\lim_{x\to \infty} \frac{x}{e^x}$

(iii) $\lim_{x\to 1} (1-x) \tan \frac{\pi x}{2}$

(iv) $\lim_{x\to 0} x^x$ 8

4. (a) Find the extreme values of the function

$$f(x) = 5x^6 + 18x^5 + 15x^4 - 10$$ 4

(b) Discuss the continuity of the function

$$f(x,y) = \frac{x^2y^2}{x^4+y^4} \text{ at } (0,0)$$ 4

(c) If $u = \tan^{-1} \frac{xy}{\sqrt{1+x^2+y^2}}$,

show that $\frac{\partial^2 u}{\partial x \partial y} = \left(1+x^2+y^2\right)^{-3/2}$ 4

8. A river is 80 feet wide. The depth d in feet at a distance x feet from one bank is given by the table

x=0	10	20	30	40	50	60	70	80
d=0	4	7	9	12	15	14	8	3

Find the area of the cross-section by –

i) Trapezoidal rule

ii) Simpson's $\frac{1}{3}$rd rule. 16

9. If u = f(x,y), $x = r\cos\theta$, $y = r\sin\theta$

Find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$. 4

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6. (a) Find the area bounded by the parabola $y^2 = 9x$ and the line $y = 3x$. 5

(b) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. 5

(c) Find the centre of gravity of the arc of the curve $x = a \sin^3\theta$, $y = a \cos^3\theta$, lying in the first quadrant. 6

7. (a) Prove that $\int_0^{\frac{\pi}{2}} \sin^m\theta \, \mathrm{Cos}^n\theta \, d\theta$

$$= \frac{\Gamma\left(\frac{m+1}{2}\right) \Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$ 8

(b) Show that $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta} \, d\theta = \frac{1}{2}\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$

$$= 2\int_0^{\infty} \frac{x^2 \, dx}{1+x^4}$$ 8

(d) If $u = \log\left[\frac{x^2+y^2}{xy}\right]$

verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$. 4

5. (a) State Euler's theorem on homogeneous functions. Verify the theorem for

$u = x^3 \log \frac{y}{x}$. 6

(b) If $u = \log \frac{x^2+y^2}{x+y}$,

prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$. 5

(c) If $u = f(r)$, $r = \sqrt{x^2+y^2}$

show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. 5

(Turn over)