Ex./CE/MATH/5/T/101/2017(OLD)(S)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester, Supplementary, Evening)

Mathematics - I (OLD)

Time : Three hours Full Marks : 100

Answer any ***six*** questions.

(Four marks are reserved for neatness)

Notations have their usual meaning.

1. Solve the following differential equations :

(a) $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$

(b) $x\ Cos^2 y\ dx - y\ Cos^2 x\ dy = 0$

(c) $\frac{dy}{dx} = Sin(x+y) + Cos(x+y)$ 5+5+6

2. Solve the following differential equations :

(a) $(x^2 + y^2)dx + (x^2 - xy)dy = 0$

(b) $\frac{dy}{dx} = \frac{y}{x} + Sin\frac{y}{x}$

(c) $x\frac{dy}{dx} - y = x\sqrt{x^2 + y^2}$ 6+5+5

(Turn over)

3. Solve the following differential equations :

(a) $(D^3 + 3D^2 + 2D)y = x^2$

(b) $(D^2 - 2D + 1)y = x^2e^{3x}$ 8+8

4. (a) Find the power series solution of the equation $\left(x^2+1\right)\dfrac{d^2y}{dx} + x\dfrac{dy}{dx} - xy = 0$ in power of x about $x=0$.

(b) Prove that $e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} t^n J_n(x)$. 9+7

5. (a) Find the power series solution of the differential equation $x^2\dfrac{d^2y}{dx^2} + \left(x^2+x\right)\dfrac{dy}{dx} + (x-9)y = 0$ in power of x about $x=0$.

(b) Prove that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}}\,\text{Sin}\,x$. 10+6

6. (a) Prove that $J_n(x) = \dfrac{1}{\pi}\int_0^{\pi} \text{Cos}\left(x\theta - x\,\text{Sin}\,\theta\right)dx$.

(b) Prove that $J_{n+1}(x) = \dfrac{2n}{\pi}J_n(x) - J_{n-1}(x)$. 10+6

7. (a) Prove that $\int_{-1}^{1} P_m(x)\,P_n(x)dx = 0$ when $m \neq n$

$= \dfrac{2}{2n+1}$ when m = n.

(b) Prove that $(2x+1)\,P_n(x) = P'_{n+1} - P'_{n-1}(x)$. 10+6

8. (a) Find the Laplace transform of

(i) e^{-3t} (Cos 4t + 3 Sin 4t).

(ii) t^2 Sin at.

(b) Solve the equation $\dfrac{d^3y}{dt^3} + 2\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 2y = 0$

where $y=1$, $\dfrac{dy}{dt} = 2$, $\dfrac{d^2y}{dt^2} = 2$, at $t=0$. 8+8

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