Ex./CE/MATH/T/122/2017(OLD)

BACHELOR OF CIVIL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - III C

Time : Three hours Full Marks : 100

Symbols and notations have their usual meanings
Use a separate Answer Scripts for each part.

PART - I (50 marks)

Answer ***q.no. 7*** and any ***four*** from the rest.

1. (a) State and prove De Moivre's theorem. 5

   (b) Show that the ratio of principal values of $(1+i)^{1-i}$ and $(1-i)^{1+i}$ is $\sin(\log 2) + i\cos(\log 2)$. 5

2. (a) If $z = \cos\theta + i\sin\theta$ and n is a positive integer, then show that

$$(1+z)^n + \left(1+\frac{1}{z}\right)^n = 2^{n+1}\cos^n\frac{\theta}{2} + \cos\frac{n\theta}{2}$$ 5

   (b) Show that the product of all the values of $\left(\sqrt{3}+i\right)^{\frac{3}{5}}$ is 8i. 3

   (c) Find arg z, where $z = 1 + i\tan\frac{3\pi}{5}$. 2

(Turn over)

3. (a) If z is a variable complex number such that mod of $\frac{z-i}{z+1}$ is k.

Show that the point z lies on a circle in the complex plane if $k \neq 1$ and z lies on a straight line if $k=1$. 5

(b) If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4+x^2+1=0$ and m is $\alpha^{2m+1} + \beta^{2m+1} + \gamma^{2m+1} + \delta^{2m+1} = 0$. 5

4. (a) State and prove D'Alembert's ratio root test for convergence or divergence series of positive terms. 5

(b) Show that the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is a monotone increasing and bounded. Is the sequence convergent ?–Justify. 5

5. (a) Find the region of convergence of the series

$$\frac{x-3}{3} + \frac{(x-3)^2}{2.3^2} + \frac{(x-3)^3}{3.3^3} + \ldots$$ 5

(b) Test the convergence of the following series

$$\left(\frac{1}{3}\right)^1 + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots$$ 5

12. (a) Show that the straight lines whose direction cosines are given by the equations $al+bm+cn=0$ and $ul^2+vm^2+wn^2 = 0$ are perpendicular or parallel according as

$a^2(w+v) + b^2(w+u) + c^2(u+v) = 0$ or

$$\frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$$ 8

(b) Find the shortest distance between the z-axis and the line

$ax+by+cz+d=0,\ a^1x + b^1y + c^1z + d^1 = 0.$ 8

13. (a) Find the equation of the cylinder whose guiding curve is $x^2+y^2=9,\ z=1$ and the fixed line is .

$$\frac{x}{2} = \frac{y}{3} + \frac{z}{-1}$$ 8

(b) If the planes through OX and OY include an angle α, show that their line of intersection lies on the cone

$$z^2(x^2+y^2+z^2) = x^2y^2\tan^2\alpha.$$ 8

—— X ——

(b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$$

8

10. (a) State and prove Cayley-Hamilton theorem. 8

(b) With the help of Cayley-Hamilton theorem find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

8

11. (a) Prove that every square matrix can be expressed as a sum of symmetric and a skew-symmetric matrix uniquely. 8

(b) Define orthogonal matrix. Show that the matrix

$$\frac{1}{3}\begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$ is orthogonal

8

6. (a) Show that the four points (–1,4,–3), (3,2,–5), (–3,8,–5), (3,2,1) are coplanar. 5

(b) Show by vector method that the angle in a semi-cricle is a right-angle. 5

7. (a) Find the unit vector perpendicular to each of $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 6\hat{j} - 2\hat{k}$. 5

(b) Prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ in a triangle ABC. 5

PART - II (50 marks)

Answer ***q.no. 8*** and any ***three*** from the rest.

8. Define rank of a matrix. 2

9. (a) Show that

$$\begin{vmatrix} (a+b)^2 & a^2 & a^2 \\ b^2 & (b+c)^2 & b^2 \\ c^2 & c^2 & (c+a)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

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