EX/CE/MATH/ T/121/2017

B.E. in Civil Engineering Examination, 2017

(1st Year, 2nd Semester)

MATHEMATICS

Paper - III C

Full Marks : 100 Time: Three hours

Use a separate answer script for each part

Symbols/Notations have their usual meanings.

Part - I (50 marks)

Answer any *five* questions. $5 \times 10 = 50$

1. Solve the following equations: (5+5)

(a)

$$(x^4 - 2xy^3)dy + (y^4 - 2x^3y)dx = 0$$

(b)

$$\frac{dy}{dx} + x\sin 2y = x\cos^2 y$$

2. Solve: (5+5)

(a)

$$(x + 3y - 2)dx - (5x + 8y - 3)dy = 0$$

(b)

$$(D^2 + 1)y = cosx$$

3. Find the series solution of Legendre equation. (10)

4. (a) Prove the recurrence formula (6+4)

$$P'_n(x) = xP'_{n-1}(x) + nP_{n-1}(x)$$

(b) Prove the following result for Bessel's function $J_n(x)$

$$(a)\ J_{\frac{1}{2}} = \frac{2}{\pi x} \sin x.$$

5. (a) Find the Fourier series for the function (5+5)

$$\begin{aligned} f(x) &= -x, \quad -\pi < x < 0 \\ &= 2x, \quad 0 < x < \pi. \end{aligned}$$

(b) For any integer n, show that

$$J_{-n}(x) = (-1)^n J_n(x).$$

6. Solve : (6+4)

(a)

$$x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$$

(b)

$$\frac{2x}{y^3}dx + \frac{y^2 - 3x^2}{y^4}dy = 0$$

7. Find the series solution near $x = 0$ of the differential equation (10)

$$x^2\frac{d^2y}{dx^2} + (x^2 + x)\frac{dy}{dx} + (x - 9)y = 0.$$

8. (a) Classify the singular points of the differential equation (2+8)

$$(x^4 - 2x^3 + x^2)y'' + 2(x - 1)y' + x^2y = 0$$

(b) Prove that

$$J_n(x)J'_{-n}(x) - J'_n(x)J_{-n}(x) = -\frac{2sin(n\pi)}{x\pi}$$

Ref. No. : **EX/CE/MATH/T/121/2017**

B.E. IN CIVIL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester)

MATHEMATICS

Paper - IIIC

Time : Three Hours **Full Marks : 100**

Use separate answer scripts for each Part

PART - II (50 Marks)

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer **any five** questions.

1. (a) If $L\{f(t)\} = F(s)$, then prove that $L\{\frac{1}{t}f(t)\} = \int_s^\infty F(s)ds$, provided the integral exists.

(b) Find the Laplace transform of $\frac{e^{-at} - e^{-bt}}{t}$.

(c) Evaluate $L^{-1}\{\frac{s}{(s^2+a^2)^2}\}$, using convolution theorem.

Or, Use Laplace transform to solve the following differential equation.

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t, y(0) = 0, y'(0) = 0.$$

4+3+3=10

2. (a) Determine the analytic function whose imaginary part is $\log(x^2 + y^2) + x - 2y$.

(b) If $f(z) = u + iv$ is holomorphic (analytic) function of $z = x + iy$, then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|f(z)\right|^2 = 4\left|f'(z)\right|^2.$$

5+5=10

3. (a) Eliminate the arbitrary functions f and g to form a partial differential equation

$$z = g(y + 6x) + f(y - 6x).$$

Or, Find the differential equation of all spheres of unit radius having their centres in the xy - plane.

(b) Solve : $\frac{\partial^3 z}{\partial^2 x \partial y} = \cos(2x + 3y)$.

5+5=10

4. Solve :

a) $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

b) $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$.

5+5=10

5. Solve :

a) $z^2(p^2x^2 + q^2) = 1$.

b) $2zx - px^2 - 2pxy + pq = 0$.

5+5=10

6. Determine the non-trivial solution of the one dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le l \ , \ t > 0,$$

subject to the following boundary and initial conditions:

$$u(0,t) = 0 \ , \ u(l,t) = 0 \ \text{ for } t > 0 \text{ and } u(x,0) = f(x) \ , \ u_t(x,0) = 0 \ \text{ for } 0 < x < l \ .$$

10

7. Solve the equation

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = 0$$

by the method of separation of variables.

10

8. Determine the non-trivial solution of the one dimensional heat equation

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le l \ , \ t > 0,$$

subject to the following boundary and initial conditions:

$$u(0,t) = 0 \ , \ u(l,t) = 0 \text{ for } t > 0 \text{ and } u(x,0) = x.$$

10