EX/CE/MATH/ T/113/2017(S)

B.CIVIL ENGG. Examination, 2017
(1ST YR, 1ST SEM,SUPPLY)
MATHEMATICS
PAPER - II C

Full Marks : 100 **Time: Three hours**

(use separate answer script)

Part - I

Answer any five questions. $5 \times 10 = 50$

1. Find the vector $\vec{d}$ which is perpendicular to both

$$\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k} \;\; ; \;\; \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k} \; and \; \vec{d} \cdot \vec{c} = 21,$$

where

$$\vec{c} = 3\hat{i} + \hat{j} - \hat{k}.$$

2. Find the unit vector vector parallel to the x-y plane and perpendicular to $4\hat{i} - 2\hat{j} + 3\hat{k}$.

3. Show that the vectors $5\hat{i} + 6\hat{j} + 7\hat{k}$, $7\hat{i} - 8\hat{j} + 9\hat{k}$ and $3\hat{i} + 20\hat{j} + 5\hat{k}$ are coplanar.

4. Find the equation of the plane passing through the points $(1, 1, 2)$ and $(2, 4, 3)$ and perpendicular to the plane

$$x - 3y + 7z + 5 = 0.$$

5. A variable plane at a distance p from the origin meets the axes at

A,B,C. Show that the locus of the centroid of the tetrahedron OABC is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$$

6. Find the equation of the image of the line

$$\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{4}$$

in the plane

$$3x+y-4z+21=0$$

7. Find the shortest distance between the lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \quad \textit{and} \quad \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$

Find also the equations and the points of intersection in which it meets the lines.

PART - II (50 marks)

Answer any ***five*** questions.

Symbols and Notations have their usual meanings.

8. (a) Prove that

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

(b) Solve the system of equations

$x + z = 0$

$3x + 4y + 5z = 0$

$2x + 3y + 4z = 1$ 4+6

9. (a) Find the eigen values and the corresponding eigen vector of the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(b) Expand by Laplace's method to prove that

$$\begin{vmatrix} a & b & c & d \\ -b & a & d & -c \\ -c & -d & a & b \\ -d & c & -b & a \end{vmatrix} = (a^2+b^2+c^2+d^2)^2$$ 6+4

10. (a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, then verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1}.

(b) If A is a real orthogonal matrix and $(I+A)$ is non singular, then prove that $(I+A)^{-1}(I-A)$ is skew symmetric. 6+4

11. (a) If $z_1^2 + z_2^2 + z_3^2 - z_1z_3 - z_3z_2 - z_1z_2 = 0$ then prove that $|z_1 - z_2| = |z_2 - z_3| = |z_3 - z_1|$.

(b) Show that the product of all the values of $\left(1 + i\sqrt{3}\right)^{3/4}$ is 8. 5+5

12. (a) Prove that the sum of 99th power of the roots of the equation $x^7 = 1$ is 0.

(b) If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$, then prove that

(i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

(ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$ 5+5

13. Test for convergence of the following series

(a) $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \ldots.$

(b) $\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \frac{11}{4.5.7} + \ldots..$

(c) $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$ 10