ii) $P(z)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$

iii) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z^2$

iv) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=nz$

v) $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0$

vi) $\frac{\partial^2 f}{\partial t^2}=c^2\left(\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}\right)$ $\frac{1}{2}\times 6=3$

Bachelor of Engineering in Chemical Engineering Examination, 2017

(2nd Year, 1st Semester)

Mathematics - IIIB (OLD)

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

(Unexplained Notations and symbols have their usual meanings)

Answer Q.No. 1 and ***two*** from the rest.

1. a) What do we usually mean by orthogonality of two real valued Riemann integrable functions f_1 and f_2 defined over a closed bounded interval [a, b] ? 2

b) Show that polynomials $P_o(x)=1$, $P_1(x)=x$ and $P_2(x)=\frac{3}{2}x^2-\frac{1}{2}$ are orthogonal on [–1, 1]. 4

c) Let $f(x)=\begin{cases}0 & -1\le x<0\\ 1 & 0\le x\le 1\end{cases}$

Find the constants C_0, C_1, C_2 such that $C_oP_o(x)+C_1P_1(x)+C_2P_2(x)$ is the Fourier expansion of f on [–1, 1]. 8

2. a) Find the Fourier series of $x - x^2$. Hence find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ (10+2)

b) Discuss the convergence of the series

i) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{2n^3+1}}$

ii) $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$ 3+3

3. a) Let $f(x) = \begin{cases} x & \text{for } 0 \le x \le \frac{L}{2} \\ L - x & \text{for } \frac{L}{2} \le x \le L \end{cases}$;

Find the Fourier sine series of f(x) on [0, L] where L is a given positive real number. 9

b) Obtain the general solution of the equation

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial x}{\partial t}$, where k is a constant,

satisfying the boundary conditions

$u(0,t) = u(L,t) = 0, t \ge 0$ ...... (i)

$u(x,0) = f(x), 0 \le x \le L$ (L is a constant) ..... (ii)

where f(x) as in 3(a). 9

b) Show that the equations xp = yq and z(xp+yq) = 2xy are compatible and solve them where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 10

c) Find the complete integral of the equation $p^2z^2 + q^2 = 1$. 4

8. a) Find the integral surface of the linear PDE

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

which contains the straight line x + y = 0, z = 1. 5

b) Find out the solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \ 0 \le x \le a, \ 0 \le y \le b$$

such that

$u_x(0,y) = u_x(a,y) = 0$ (a, b are given constants)

$u_y(x,0) = 0$, $u_y(x,b) = f(x)$, where f(x) is a given function. 8

c) Find a complete integral of the equation

$$pq = 1$$ 2

d) Classify the following PDE's

i) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 1$

PART - II

Answer **Q.No. 5** and ***two*** from the rest.

5. a) Solve $(y^4 - 2x^3y)dx + (x^4 - 2xy^3)dy = 0$ 4

   b) Solve $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = xe^{-2x}$ 6

   c) Find all solution of

   $$\frac{dy}{dx} = y^{1/3}, y(0) = 0$$ 4

6. a) Find a power series solution of the intial-value problem.

   $(x^2 - 1)y'' + 3xy' + xy = 0, \quad y(0) = 4, \quad y'(0) = 6.$ 5

   b) Write down the definition of regular and irregular singular points. 2

   c) Find an integrating factor of

   $$(x^4y^2 - y)dx + (x^2y^4 - x)dy = 0$$

   and hence solve it. 8

7. a) Let $P_x(x)$ denote the legendre polynomial of degree n. Prove that

   $$x^4 = \frac{1}{5}P_0(x) + \frac{4}{7}P_2(x) + \frac{8}{35}P_4(x)$$

4. a) State and prove Alternating series test. 5

   b) State limit coparison test for two series of positive real numbers. 2

   c) Obtain the Fourier series of the function

   $$f(x) = \begin{cases} x - \pi, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

   Hence deduce that

   $$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$ 9+2

[Turn over