

**Bachelor of Chemical Engineering Examination, 2017**

(2<sup>nd</sup> Year, 2<sup>nd</sup> Semester)

**INTRODUCTION TO TRANSPORT PHENOMENA**

Time: Three Hours

( 50 marks for each part )

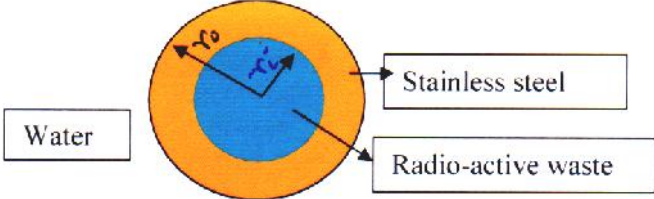
Full Marks: 100

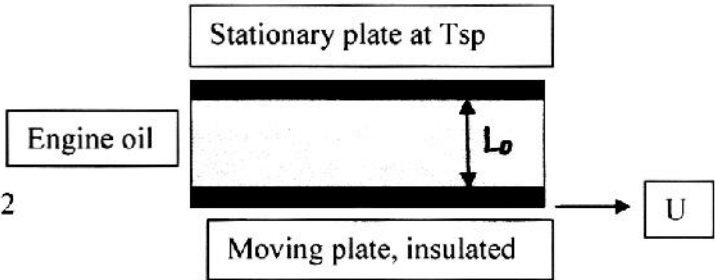
Use a separate Answer-Script for each part

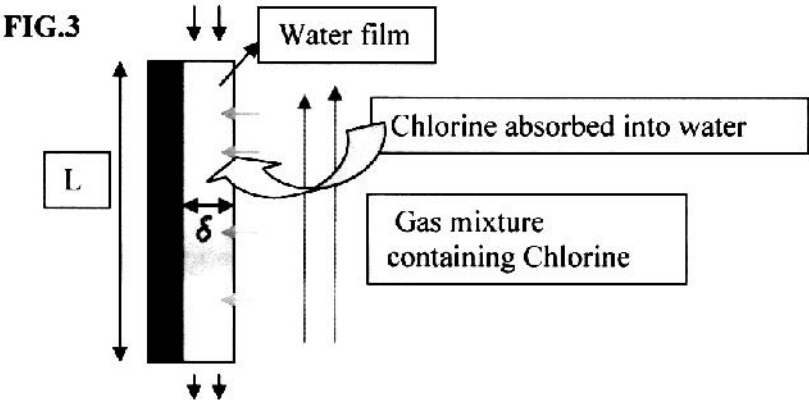
**PART - I**

Answer any two questions

State all the assumptions; Assume missing data (if any)

No. of questions		Marks
1(a)	<p>Radioactive wastes (<math>k_{rw}=20\text{W/m.K}</math>) are stored in a spherical stainless steel (<math>k_{ss}=15\text{ W/mK}</math>) container of inner and outer radii equal to <math>r_i=0.5\text{ m}</math>, <math>r_o=0.6\text{ m}</math>. Heat is generated volumetrically within the wastes at a uniform rate <math>q_g=10^5\text{ W/m}^3</math> and the outer surface of the container is exposed to water for which <math>h=1000\text{W/(m}^2\text{K)}</math> and temperature is <math>25^\circ\text{C}</math>.</p> <p>(i) Obtain an expression for temperature distribution inside the radioactive wastes. Express your results in term of <math>r_i</math>, <math>T_{si}</math>, <math>k_{rw}</math> and <math>q_g</math></p> <p>(ii) Evaluate the temperature at <math>r=0</math></p> <p>(ii) Evaluate the steady state inner surface temperature <math>T_{si}</math> and steady state outer surface temperature <math>T_{so}</math>.</p> <div style="text-align: center;">  </div> <p>FIG.1</p>	(13)
1(b)	<p>An ethanol-water vapor mixture is being rectified by contact with an ethanol-water liquid solution. The ethanol (A) is transferred from the liquid to the vapor phase and the water (B) is transferred in the opposite direction (from the vapor to the liquid phase). Both components are diffusing through a gas film of <math>0.1\text{ mm}</math> thick. The temperature is <math>368\text{ K}</math> and the pressure is <math>1.013\times 10^5\text{ Pa}</math>. At that temperature the latent heat of vaporization of the alcohol and water are <math>1.122\times 10^6\text{ J/kg}</math> and <math>2.244\times 10^6\text{ J/kg}</math>, respectively. The mole fraction of ethanol is <math>0.8</math> on one side of the gas film and <math>0.2</math> on the other side of the film. Considering steady state operation</p> <p>(i) Derive the expressions for flux of ethanol (A) and flux of water (B).</p> <p>(ii) Calculate the rate of diffusion of ethanol and of water in <math>\text{kg/s}</math> through one square meter of area. The diffusion coefficient <math>D_{AB} = 6\times 10^{-6}\text{ m}^2/\text{s}</math>.</p> <p>(iii) Derive the expression for concentration profile of ethanol across the gas film.</p>	(12)

No. of questions		Marks
2.	<p>Consider Couette flow with heat transfer for which the lower plate (<math>y=0</math>) moves with a speed of <math>U = 15</math> m/s and is perfectly insulated. The upper plate (<math>y=L_0</math>) is stationary and is maintained at <math>T_{sp}=40^\circ\text{C}</math>. The plates are separated by a distance <math>L_0= 5</math> mm, which is filled with an engine oil of viscosity <math>\mu=0.8</math> N s/m<sup>2</sup> and thermal conductivity <math>k_0=0.145</math> W/mK.</p> <p>(i) Derive the velocity distribution  (ii) Obtain an expression for the temperature distribution inside the fluid engine oil in terms of the plate speed <math>U</math>, the temperature of the stationary plate (<math>T_{sp}</math>) and the oil parameters (<math>\mu</math>, <math>k_0</math>, <math>L_0</math>).  (iii) Calculate the temperature at the lower surface of the oil film <math>T_{L_0}</math>. For incompressible Newtonian fluid flow with constant properties, Navier stokes equation &amp; thermal energy balance equations are as follows</p> $\rho \left[ \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right] = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{V}$ $\rho C_p \left[ v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} \right] = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \mu \Phi + q_x$ $\mu \Phi = \mu \left[ \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)^2 + 2 \left[ \left( \frac{\partial v_x}{\partial x} \right)^2 + \left( \frac{\partial v_y}{\partial y} \right)^2 \right] \right]$ <div style="text-align: center;">  </div>	(10+15)

No. of questions		Marks
3.	<p>Consider the absorption of Chlorine (A) in to a water (B). A thin film of a water (of thickness <math>\delta</math>) flows along (z) a vertical wall of length L (=13 cm) and width W(=10 cm). The absorbing liquid (A) water is, flowing with an average velocity of 20 cm/s. It absorbs chlorine from the gas mixture (refer to figure 3). The saturation concentration of chlorine in water is 0.823 g <math>\text{Cl}_2</math>/100g water. The concentration at the gas liquid interface may be assumed to be equal to saturation concentration. The diffusion coefficient <math>D_{\text{Cl}_2, \text{H}_2\text{O}} = 1.26 \times 10^{-5} \text{ cm}^2/\text{s}</math> in the liquid phase. Consider that chlorine penetrates only a short distance in to the water film (before it is carried by the flow) due to slow rate of diffusion.</p> <p>(i) Derive the expression for steady state velocity profile of the falling film of liquid B (WATER).</p> <p>(ii) Derive the expression for the average velocity of water</p> <p>(iii) Calculate the value of <math>\delta</math></p> <p>(iv) Derive the steady state concentration profile of chlorine in water film.</p> <p>(v) What is the total absorption rate of Chlorine into water?</p> <p>(vi) Write the expression for mass transfer coefficient.</p> <p><b>FIG.3</b></p>  <p>The diagram shows a vertical wall of length L. A thin film of water of thickness <math>\delta</math> flows down the wall. A gas mixture containing chlorine is on the right, and chlorine is being absorbed into the water film. Arrows indicate the flow of water down the wall and the absorption of chlorine from the gas into the liquid.</p>	(10)+(15)

[ Turn over

No. of questions		Marks
4. (a)	<p>(i) Write the conditions of Reynolds analogy for flow over a flat plate.</p> <p>(ii) As a means of preventing ice formation on the wings of a small, private aircraft, it is proposed that electric resistance heating elements be installed within the wings. To determine representative power requirements, consider normal flight conditions for which the plane moves at 100m/s in air that is at a temperature of -23°C and has properties of <math>k=0.022 \text{ W/(mK)}</math>, <math>Pr \sim 1</math>, <math>\nu=16 \times 10^{-6} \text{ m}^2/\text{s}</math>. If the characteristic length of airfoil <math>L=2\text{m}</math> and wind tunnel measurement indicates an average friction coefficient <math>\overline{C_f}=0.0025</math> for the nominal conditions, what is the average heat flux needed to maintain a surface temperature of <math>T_s=5^\circ\text{C}</math>?</p>	(4+6)
4(b)	<p>A Newtonian fluid of constant density and viscosity is in a cylindrical container of radius <math>R</math>. The container is caused to rotate about its own axis at an angular velocity of <math>\Omega</math>. The cylinder axis is vertical so that <math>g_r=g_\theta=0</math> and <math>g_z=-g</math>.</p> <p>(i) Derive the steady state velocity (<math>v_\theta</math>)</p> <p>(ii) Derive the steady state pressure profile</p> <p>(iii) Prove that the shape of the free surface is described by an equation of a parabola.</p> <p>The equation of continuity and Navier stokes equation in cylindrical coordinate system</p> $\frac{1}{r} \frac{\partial(rv_r)}{\partial r} + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$ $\rho \left( \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \right) = -\frac{\partial p}{\partial r}$ $+ \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial(rv_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] + \rho g_r$ $\rho \left( \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta}$ $+ \mu \left[ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial(rv_\theta)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_\theta}{\partial z^2} \right] - \rho g_\theta$ $\rho \left( \frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z}$ $+ \mu \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho g_z$	(5+5+5)

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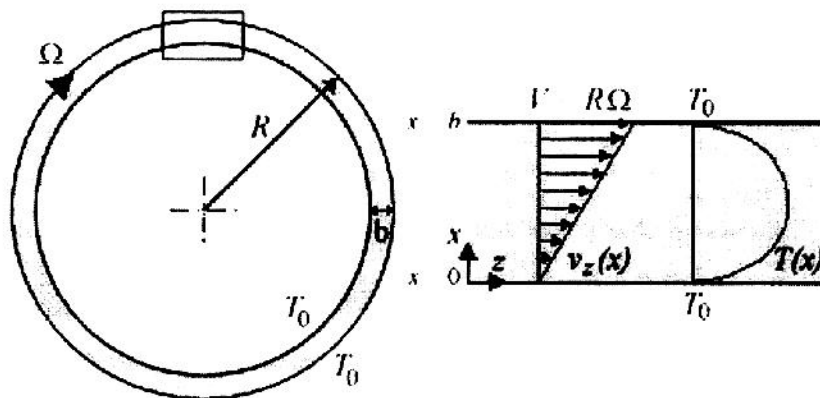
Full Marks: 100

**Part II**

*Answer any two questions. Each question carries 25 marks.*

[1]

An oil (of viscosity  $m$  and thermal conductivity  $k$ ) acts as a lubricant between two coaxial cylinders. The inner cylinder is stationary and the outer cylinder of radius  $R$  rotates at an angular velocity  $\Omega$ . The clearance between the cylinders is  $b$ , which is small compared to the radii of the cylinders; so, curvature effects can be neglected and the cylindrical system can be approximated by a plane narrow slit (to be solved in Cartesian coordinates) as shown in the figure. Derive an expression for the maximum temperature in the lubricant if both cylinders are at temperature  $T_0$ . Neglect the temperature dependence of  $m$  and  $k$ , but explicitly take into account the heat generated by viscous dissipation.

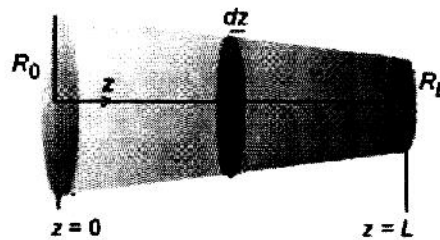


*Temperature profile for viscous heat generation. The rectangular section in the flow between two coaxial cylinders can be approximated by the plane narrow slit on neglecting the curvature of the bounding surfaces.*

[2]

A fluid (of constant density  $\rho$ ) is in incompressible, laminar flow through a tube of length  $L$ . The radius of the tube of circular cross section changes linearly from  $R_0$  at the tube entrance ( $z = 0$ ) to a slightly smaller value  $R_L$  at the tube exit ( $z = L$ ).

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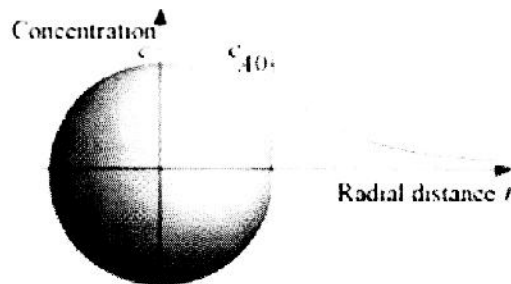


Using the lubrication approximation, determine the mass flow rate vs. pressure drop ( $w$  vs.  $\Delta P$ ) relationship for a Newtonian fluid (of constant viscosity  $\mu$ ).

*Hint: Where a flow between non-parallel surfaces is treated locally as a flow between parallel surfaces is commonly called the **lubrication approximation** because it is often employed in the theory of lubrication. The lubrication approximation, simply speaking, is a local application of a one-dimensional solution and therefore may be referred to as a quasi-one-dimensional approach.*

[3]

A solid sphere (of radius  $R$  and density  $r$ ) made of substance A (of molecular weight  $M$ ) is suspended in a liquid B. Solid A undergoes a first-order homogeneous chemical reaction with rate constant  $k_1$  being slightly soluble in liquid B. Let  $c_{A0}$  be the molar solubility of A in B, and  $D_{AB}$  be the diffusivity of A in B.



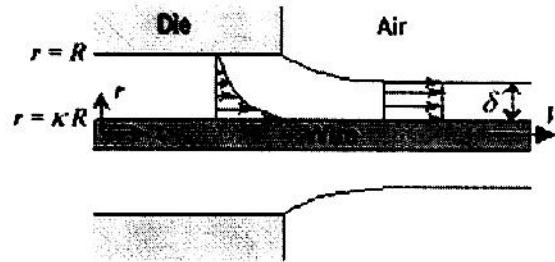
a) Establish the concentration profile for A at steady state (i.e., when the mass diffusion is in exact balance with the chemical reaction).

b) Using a quasi-steady-state approach, derive an expression for the time  $t$  required for the sphere radius to decrease from an initial radius  $R_0$  to  $R$ .

[4]

A wire - coating die essentially consists of a cylindrical wire of radius  $\kappa R$  moving horizontally at a constant velocity  $V$  along the axis of a cylindrical die of radius  $R$ . If the pressure in the die is uniform, then the polymer melt (which may be considered a non-Newtonian fluid described by the power law model and of constant density  $\rho$ ) flows through the narrow annular region solely by the drag due to the axial motion of the wire

(which is referred to as 'axial annular Couette flow'). Neglect end effects and assume an isothermal system. The power law fluid follows:  $\tau_{rz} = m \left( -\frac{dv_z}{dr} \right)^n$



- Establish the expression for the steady-state velocity profile in the annular region of the die. Simplify the expression for  $n = 1$  (Newtonian fluid).
- Obtain the expression for the mass flow rate through the annular die region. Simplify the expression for  $n = 1$  and  $n = 1/3$ .
- Estimate the coating thickness  $\delta$  some distance downstream of the die exit.
- Find the force that must be applied per unit length of the wire.

[5]

Starting with a differential energy balance in 2 dimensions, develop an expression for the integral energy equation of the boundary layer for laminar flow over a heated flat plate for constant fluid properties and constant free-stream temperature,  $T_\infty$ . Derive expressions for the temperature distribution and thermal boundary layer thickness. Show that

$$\frac{\delta_{\text{thermal}}}{\delta_{\text{hydrodynamic}}} = \frac{1.026}{Pr^{1/3}}$$

### Continuity Equation

*Cylindrical*

$$\frac{\partial \rho}{\partial t} + \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \rho V_r) + \frac{1}{r} \frac{\partial}{\partial \theta} (\rho V_\theta) + \frac{\partial (\rho V_z)}{\partial z} \right\} = 0$$

### Navier Stokes Equations

*Cartesian*

$$x: \quad \rho \left\{ \frac{\partial V_x}{\partial t} + V_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} \right\} = \rho g_x - \frac{\partial p}{\partial x} + \mu \left\{ \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial y^2} + \frac{\partial^2 V_x}{\partial z^2} \right\}$$

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$$y: \quad \rho \left\{ \frac{\partial V_y}{\partial t} + V_x \frac{\partial V_y}{\partial x} + V_y \frac{\partial V_y}{\partial y} + V_z \frac{\partial V_y}{\partial z} \right\} = \rho g_y - \frac{\partial p}{\partial y} + \mu \left\{ \frac{\partial^2 V_y}{\partial x^2} + \frac{\partial^2 V_y}{\partial y^2} + \frac{\partial^2 V_y}{\partial z^2} \right\}$$

$$z: \quad \rho \left\{ \frac{\partial V_z}{\partial t} + V_x \frac{\partial V_z}{\partial x} + V_y \frac{\partial V_z}{\partial y} + V_z \frac{\partial V_z}{\partial z} \right\} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left\{ \frac{\partial^2 V_z}{\partial x^2} + \frac{\partial^2 V_z}{\partial y^2} + \frac{\partial^2 V_z}{\partial z^2} \right\}$$

*Cylindrical*

*r:*

$$\rho \left\{ \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} - \frac{V_\theta^2}{r} + V_z \frac{\partial V_r}{\partial z} \right\}$$

$$= \rho g_r - \frac{\partial p}{\partial r} + \mu \left\{ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r V_r) \right) + \frac{1}{r^2} \frac{\partial^2 V_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial V_\theta}{\partial \theta} + \frac{\partial^2 V_r}{\partial z^2} \right\}$$

*$\theta$ :*

$$\rho \left\{ \frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{V_r V_\theta}{r} + V_z \frac{\partial V_\theta}{\partial z} \right\}$$

$$= \rho g_\theta - \frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left\{ \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r V_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial V_r}{\partial \theta} + \frac{\partial^2 V_\theta}{\partial z^2} \right\}$$

*$z$ :*

$$\rho \left\{ \frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right\} = \rho g_z - \frac{\partial p}{\partial z} + \mu \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2} \right\}$$

## Mass Diffusion Equation

*Spherical Coordinates*

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( D_{AB} r^2 \frac{\partial C_A}{\partial r} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \phi} \left( D_{AB} \frac{\partial C_A}{\partial \phi} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( D_{AB} \sin \theta \frac{\partial C_A}{\partial \theta} \right) + N_A \dot{=} \frac{\partial C_A}{\partial t}$$