***OR*-12**

Obtain the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial v^2}$$

under the following conditions.

i) $u(0,t) = u(2,t) = 0$

ii) $u(x,0) = \text{Sin}^3 \frac{\pi x}{2}$

iii) $u_t(x,0) = 0$ 10

Bachelor of Engineering in Chemical Engineering Examination, 2017

(2nd Year, 1st Semester)

Mathematics - III

Time : Three hours Full Marks : 100

(50 marks for each part)

Use a separate Answer-Script for each part

PART - I

(Unexplained Notations and symbols have their usual meanings)

Answer Q.No. 1 and ***two*** from the rest.

1. a) What do we usually mean by orthogonality of two real valued Riemann integrable functions f_1 and f_2 defined over a closed bounded interval [a, b] ? 2

b) Show that polynomials $P_o(x) = 1$, $P_1(x) = x$ and $P_2(x) = \frac{3}{2}x^2 - \frac{1}{2}$ are orthogonal on [–1, 1].

Let $f(x) = \begin{cases} 0 & -1 \le x < 0 \\ 1 & 0 \le x \le 1 \end{cases}$

c) Find the constants C_0, C_1, C_2 such that $C_oP_o(x) + C_1P_1(x) + C_2P_2(x)$ is the Fourier expansion of f on [–1, 1]. 8

[Turn over

2. a) What do we usually mean by Fourier series of a Riemann integrable function f on $[-\pi, \pi]$? 3

b) Find the Fourier series of $x - x^2$ Hence find the sum of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots$ (8+2)

c) What do we usually mean by

i) Fourier Sine series,

ii) Fourier Cosine series of a Riemann integrable function over $[0, \pi]$? Find Fourier co-efficients in each case.

5

3. a) Suppose L is a positive real number. Write the orthogonal system considered for determining the

i) Fourier series,

ii) Fourier cosine series,

iii) Fourier sine series,

of a Riemann integrable function f(x) respectively over [–L, L], [0, L], [0, L]. 2+2+2

b) In order to obtain the general solution of the equation

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial x}{\partial t}$, where k is a constant,

iv) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$

v) $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

vi) $\frac{\partial^2 f}{\partial t^2} = c^2\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$ $\frac{1}{2}\times 6 = 3$

11. a) Find out the solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \le x \le a, \quad 0 \le y \le b$$

such that

$u_x(0, y) = u_x(a, y) = 0$ (a, b are given constants)

$u_y(x, 0) = 0$, $u_y(x, b) = f(x)$, a given function. 8

b) Find a complete integral of the equation pq = 1. 2

12. A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time t = 0, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x, t)$. 10

b) Find the Laplace transforms of

i) t^3e^{-3t},

ii) $2^t + \frac{\text{Cos}2t + \text{Cos}3t}{t} + t\sin t$ 1+4

c) i) Use convolution theorem to find the inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$.

ii) Find the inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$

(a is a constant) 4+4

5. a) Use the Method of Laplace transform to solve

i) $t\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + ty = \cos t$ given that $y(0) = 1$

ii) $\frac{d^2y}{dt^2} + 9y = \text{Cos}2t$ given that $y(0) = 1$, $y\left(\frac{\pi}{2}\right) = 1$.

4+4

b) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 < x < 2 \\ 0 & \text{for } x > 2 \end{cases}$$ 4

c) Find the inverse z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$ 6

PART - II

(Unexplained Notations and symbols have their usual meanings)

Answer ***any five*** questions.

6. a) Solve $(y^4 - 2x^3y)dx + (x^4 - 2xy^3)dy = 0$ 5

b) If $M(x,y)dx + N(x,y)dy = 0$, $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, show that $\mu = e^{\int f(x)dn}$ is an integration factor of the equation. Hence show that $e^{\int Pdx}$ is an integration factor of $\frac{dy}{dx} + P(x)y = \theta(x)$ 3+2

7. a) Solve $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4y = xe^{-2x}$ 6

b) Find all solution of

$\frac{dy}{dx} = y^{1/3}, y(0) = 0$ 4

8. a) Find a power series solution of the intial-value problem.

$(x^2-1)y'' + 3xy' + xy = 0$, $y(0) = 4$, $y'(0) = 6$. 5

b) Find the roots of the indicial equation of the different equation

$2x^2y'' + xy' + (x^2-3)y = 0$ 3

[Turn over

c) Write down the definition of regular and irregular singular points. 2

9. a) Show that the equations xp = yz and z(xp+yz)=2xy are compatible and solve them where $p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}$. 7

b) Find the complete integral of the equation $p^2z^2+z^2=1$. 3

10. a) Find a complete integral of the equation

$$(p+q)(z-xp-yq)=1$$ 2

b) Find the integral surface of the linear PDE

$$x(y^2+z)p-y(x^2+z)q=(r^2-y^2)z$$

which contains the straight line x + y = 0, z = 1. 5

c) Classify the following PDE's

i) $\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2=1$

ii) $P(z)\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$

iii) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=z^2$

satisfying the boundary conditions

$u(0,t)=u(L,t)=0, t\geq 0$ ...... (i)

$u(x,0)=f(x), 0\leq x\leq L$ (L is a constant) ..... (ii)

where $f(x)=\begin{cases} x & \text{for } 0\leq x\leq \frac{L}{2} \\ L-x & \text{for } \frac{L}{2}\leq x\leq L \end{cases}$;

one will obtain at certain stage the general solution as $u(x,t)=\sum_{r=1}^{\infty}B_r e^{(-r^2\pi^2kt)/L^2}\operatorname{Sin}\frac{r\pi x}{L}$ satisfying the boundary conditions given by (i) where the constants B_r's must be chosen to satisfy boundary conditions given by (ii). Find B_r for all r and obtain the final solution. 12

4. a) Using definition of Laplace transform find L(F(t)) in each of the following cases :

i) $F(t)=\begin{cases} \operatorname{Cos}\left(t-\frac{2\pi}{3}\right), & t>\frac{2\pi}{3} \\ 0 & t<\frac{2\pi}{3} \end{cases}$.

ii) F(t) = sin at (a is a constant) 3+2