Ex/CHE/Math/T/226/2017(Old)

Bachelor of Engineering in Chemical Engineering Examination, 2017

(2nd Year, 2nd Semester, Old)

Mathematics IVB

Time : Three hours | Full Marks : 100

Answer any ***five*** questions

1. a) If a function $f(z) = u(x, y) + iv(x, y)$ is differentiable at a point z = x + iy prove that it satisfies Cauchy-Riemann equations. 10

   b) Determine the analytic function $f(z) = u + i\nu$ where

   $$u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$ 10

2. a) If f(z) is a regular function of z, prove that

   $$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ 8

   b) State and prove Cauchy's integral formula. 12

3. a) Determine the residue at the singularities of the function

   $$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+1)}$$ 8

   b) Evaluate $\int_C \frac{zdz}{(z-1)(z-2)^2}$ where C is the circle 12

4. a) Evaluate the surface integral $\int_C \vec{F} \cdot \vec{n}\, ds$ where $\vec{F} = 2x^2z\vec{i} + y^2\vec{j} - 2yz\vec{k}$ and S is given by the surface of the unit cube x = 0, x = 1 ; y = 0, y = 1 ; z = 0, z = 1 ; also $\vec{n}$ is the otward drawn unit normal. 12

   b) If $\vec{F} =$ grad $(x^3 + y^3 + z^3 - 3xyz)$, then show that the vector is irrotational. 8

[Turn over

5. a) If f(t) of exponential order γ as $t \to \infty$ and is piecewise continuous over every finite interval $t \geq 0$, of then show that Laplace transform of f(t) exists for $s > \gamma$. 10

b) Solve by Laplace transform method :

$$f(t) = t + 2\int_0^t f(u)\cos(t-u)du$$ 10

6. a) Find Fourier transform of f(x) defined by

$$f(x)\begin{cases} 1, & |x| \leq a \\ 0, & |x| > a \end{cases}$$

and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$ 10

b) Using z-transform, solve

$$u_{n+2} + 4u_{n+1} + 3u_n = 3^n$$

with $u_o = 0$, $u_1 = 1$. 10

7. a) Define covarient and contravarient tensors of order two. A covarient tensor has components xy, $2y - z^2$, xz in rectangular co-ordinates. Find its covarient components is spherical co-ordinates. 12

b) Show that the Kronecker delta is a mixed tensor of order two. 8