12. a) State and prove Trapezoidal rule.

b) The curve $y = x^2 - 1$ is rotated about the x-axis through 360^o. Find the volume of the solid generated when the area contained between the curve and the x-axis is rotated about the x-axis by 360^o. 5+5

13. a) Let $f(x) = [\,x\,]$, $x \in [\,0, 3\,]$. Prove that f is integrable on $[\,0, 3\,]$. Also Evaluate $\int_0^3 f(x)\,dx$.

b) State 2nd mean value theorem :- Bonnet's form. 7+3

17. State and prove fundamental theorem of Integral calculus. 10

Bachelor of Engineering in Chemical Engineering Examination, 2017

(1st Year, 1st Semester)

Mathematics

Time : Three hours Full Marks : 100

(50 marks for each Part)

Use separate answer script for each Part

PART - I

Answer ***any five*** questions

1. a) If $ax^2 + 2hxy + by^2 = 1$ show that

$$\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$$

b) If $y = x^{2n}$, where n is a positive integer, show that

$$y_n = 2^n \{1 \cdot 3 \cdot 5 \,.....\, (2n-1)\}\, x^n.$$ 5+5

2. Find y_n for x = 0 when $y = e^{a \sin^{-1} x}$ 10

3. State and prove mean value Theorem. Give a geometrical interpretation of mean value theorem. 10

4. Expand $(\sin^{-1} x)^2$ in a power series of ascending powers of x [use method of differential equation] 10

[Turn over

5. Evaluate 5+5

i) $y = \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

ii) If $\underset{x \to 0}{\text{Lt}} \frac{\sin 2x + a \sin x}{x^3}$ be finite, find the value of 'a' and the limit.

6. If $f(x, y) = (x^2 + y^2)^{1/3}$. Use Euler's theoren to find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial t}{\partial y}$ and hance prove that

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 t}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = -\frac{2}{x} f$$ 10

7. Test the convergence of the following series 5+5

i) $\frac{1^2}{2} + \frac{2^2}{2^2} + \frac{3^2}{2^3} + \frac{4^2}{2^4} + \cdots$

ii) $\sum u_n$ where $u_n = \left\{\left(\frac{n+1}{n}\right)^{n+1} - \left(\frac{n+1}{n}\right)\right\}^{-n}$

PART - II

Answer ***any five*** questions

8. a) Evaluate the integral over the given region D.

$$\iint_D e^{x/y} dA, \ D = \{(x, y) \mid 1 \le y \le 2, \ y \le x \le y^3\}$$

b) Evaluate $\iint_R x \cos^2(y) dA, \ R = [-2, 3] \times [0, \pi/2]$ 5+5

9. a) Show that the 2nd mean value theorem (Weierstrass' form) is applicable to $\int_a^b \frac{\sin x}{x} dx$ where $0 < a < b < \infty$.

Also show that $\left| \int_a^b \frac{\sin x}{x} dx \right| < 4/a$.

b) State first mean value theorem of Integral calculus.

c) If $f : [a, b] \to \mathbb{R}$ be continuous on [a, b] and $\int_a^b f(x) dx = 0$. Prove that $\exists$ at least a point $c \in [a, b]$ such that $f(c) = 0$. 5+2+3

10. State and prove that 2nd mean value theorem :- 'Weierstrass' form'. 2+8

11. a) Prove that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx, m > 0, n > 0.$

b) Show that $\int_0^1 \frac{1}{(1-x^n)^{1/n}} dx = \frac{\pi}{n} \operatorname{cosec} \frac{\pi}{n}, \ n > 1.$ 5+5

[Turn over