Ref. No. : EX/CHE/MATH/T/123/2017

B. E. CHEM 1ST YR, 2ND SEM EXAM, 2017
MATHEMATICS – II

TIME: Three Hours **Full Marks: 100**

1. **50 marks for each Group**
2. **Use separate answer script for each group**

GROUP – A

(Answer **any five** questions and **bold letters** indicate the vector quantity)

1. a) Show that $|\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C}| \leq |\boldsymbol{A}| + |\boldsymbol{B}| + |\boldsymbol{C}|$.

   b) Prove that the diagonals of a parallelogram bisect each other.

   c) A force given by $\boldsymbol{F} = 3\boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ is applied at the point (1, -1, 2). Find the moment of $\boldsymbol{F}$ about the point (2, -1, 3).

   d) Find a set of vectors reciprocal to the set $2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{i}$-$\boldsymbol{j}$-$2\boldsymbol{k}$ and $-\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$.

   $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$

2. a) Given the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$, find
   i) The curvature k. ii) The torsion τ.

   b) $\varphi(x, y, z) = xy^2z$ and $\boldsymbol{A} = xz\boldsymbol{i} - xy^2\boldsymbol{j} + yz^2\boldsymbol{k}$ find $\frac{\partial^3}{\partial x^2\,\partial x}(\varphi\boldsymbol{A})$ at the point (2, -1, 1).

   8+2

3. a) Determine the constant "a" so that the vector $\mathbf{V} = (x + 3y)\mathbf{i} + (y - 2z)\mathbf{j} + (x + az)\mathbf{k}$ is solenoidal.

   b) Two rectangular xyz and XYZ coordinate systems having the same origin are rotated with respect to each other. Derive the transformation equations between the coordinates of a point in the two systems.

   3+7

4. a) If $\boldsymbol{F}$ is a conservative field, prove that $curl\ \boldsymbol{F} = \nabla \times \boldsymbol{F} = 0$.

   b) Conversely, if $\nabla \times \boldsymbol{F} = 0$, prove that $\boldsymbol{F}$ is conservative.

   5+5

5. a) If $\boldsymbol{F} = 4xyz\,\boldsymbol{i} - y^2\,\boldsymbol{j} + yz\,\boldsymbol{k}$, evaluate $\iint \boldsymbol{F}.\boldsymbol{n}\, ds$ where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.

[Turn over

b) State The Divergence Theorem Of Gauss.

8+2

6. Prove Stokes' theorem.

10

7. a) A covariant tensor has components $xy, 2y - z^2, xz$ in rectangular coordinates. Find its covariant components in spherical coordinates.

b) Show that $\frac{\partial A_p}{\partial x^q}$ is not a tensor even though A_p is a covariant tensor of rank one.

7+3

Group-B

(Answer any five questions)

10x5=50

1. (a) Prove that $l^2 + m^2 + n^2 = 1$. 4+6=10

   (b) If a line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, then prove that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}.$$

2. (a) Define direction cosines (d.cs.) of a straight line. 4+6=10

   (b) Show that the straight lines whose d.cs. are given by $al + bm + cn = 0, fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.

3. (a) Show that the necessary condition for the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ to represent two planes is $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$.

   5

   (b) A variable plane passes through a fixed point (α, β, γ) and meets the axes of reference in A,B and C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the coordinate planes is $\frac{\alpha}{x} + \frac{\beta}{y} + \frac{\gamma}{z} = 1$. 5

4. (a) Investigate for what values of α and β the simultaneous equations

$$x + y + z = 6,$$
$$x + 2y + 3z = 10,$$
$$x + 2y + \alpha z = \beta,$$

   have (i) no solution, (ii) a unique solution and (iii) infinitely many solution. 5

   (b) Show that the shortest distance between any two opposite edges of the tetrahedron formed by the planes $y + z = 0, z + x = 0, x + y = 0, x + y + z = c$ is $\frac{2c}{\sqrt{6}}$ and the three lines of shortest distance intersect at the point $x = y = z = -c$. 5

5. (a) Define unitary matrix and show that $A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is an unitary matrix. 4

   (b) Show that every square matrix with complex entries can be uniquely expressed as the sum of a *Hermitian* matrix and a *skew Hermitian* matrix. 3

   (c) Show that $\det A = (x - y)(x - z)(x - w)(y - z)(y - w)(z - w)$, where 3

$$A = \begin{bmatrix} 1 & x & x^2 & x^3 \\ 1 & y & y^2 & y^3 \\ 1 & z & z^2 & z^3 \\ 1 & w & w^2 & w^3 \end{bmatrix}$$

6. (a) Define rank of a matrix and using Echelon form show that rank of matrix A is 2, where 4

$$A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

(b) Let A, B be two matrices of order 10 and $r(A) = 9$ and $r(B) = 4$. Then prove that $r(AB)$ is either 3 or 4. 2

(c) Using Cramer's rule solve the following system of equations

$$2x - y + 3z = 8,$$

$$-x + 2y + z = 4,$$

$$3x + y - 4z = 0.$$ 4

7. (a) Prove that eigenvalues of a Hermitian matrix are real. 3

(b) Determine the eigenvalues of the matrix, $M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{bmatrix}$. 2

(c) State Cayley-Hamilton theorem and using this theorem find A^{-1}, where 5

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

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