9. a) Explain the concept of Riemann integrability of a bounded function f(x) on [a, b].

b) Show that the constent function f(x) = K is R-integrable and evaluate it.

c) State and prove the necessary condition for Riemann integrability of a bounded function f(x) on [a, b]. 2+3+5

10. a) Calculate by Simpson's one-third rule the value of the integral $\int_0^1 \frac{x\,dx}{1+x}$, correct upto three significant figures, by taking six intervals.

b) If $f:[a,b]\to\mathbb{R}$ is monotone on [a, b], then prove that f is Riemann integrable on [a, b]. 6+4

11. a) Evaluate the integral $\int_0^1 \frac{dx}{1+x^2}$ using the trapezoidal rule with 4 equal subintervals.

b) Evaluate $\iint_R \sin(x+y)\,dxdy$

over $R:\left\{0\le x\le \pi/2;\ 0\le y\le \pi/2\right\}$. 6+4

12. a) Find the area of the loop formed by the curve

$$a^2y^2 = x^3(2a - x).$$

b) Find the area of the region founded by the curve $y = x^2$ and $x = y^2$. 6+4

Ex/CHE/MATH/T/114/2017 **(Old)**

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2017

(1st Year, 1st Semester) **Old**

MATHEMATICS - IB

Time : Three hours Full Marks : 100

(50 marks for each Group)

Use separate answer script for each Group

Answer ***any ten*** questions 10×10=100

1. a) Prove that $\lim_{x\to 0} \frac{1}{e^{1/x}+1}$ does not exist.

b) If $y=\cos\left(m\sin^{-1}x\right)$, then prove that

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}+\left(m^2-n^2\right)y_n=0.$$

Hence find y_n for x = 0. 4+6

2. a) Prove that between any two real roots of $e^x\sin x = 1$, then is at least one real root of $e^x\cos x = -1$.

b) If $y = x^{n-1}\log x$, then prove that $y_n = \frac{(n-1)!}{x}$ 5+5

3. a) State Rolle's theorem. Are the three conditions of Rolle's theorem necessary ? Justify your answer.

b) Determine the values of a, b, c, so that

$$\frac{ae^x - b\cos x + ce^{-x}}{x\sin x}\to 2 \text{ as } x\to 0$$ 5+5

[Turn over

4. a) Apply mean value theorem to prove that

$$\frac{x}{1+x} < \log(1+x) < x, \text{ for all } x > 0.$$

b) Suppose the function f(x, y) defined by

$$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & x^2+y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$$

Verify $f_{xy}(0,0)$ and $f_{xy}(0,0)$ are equal or not. 5+5

5. a) Define homogeneous function.

If $u = \sin^{-1}\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{12}\tan u = 0$$

b) If $u = e^{xyz}$, then find the value of $\dfrac{\partial^3 u}{\partial x \partial y \partial z}$. 6+4

6. a) Examine the continuity of the function

$$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x-y}, & \text{when } x \neq y \\ 0, & \text{when } x = y \end{cases}$$

b) Show that $\lim_{(x,y)\to(0,0)} f(x,y) = 0,$

where $f(x,y) = x\sin\frac{1}{y} + y\sin\frac{1}{x}$, when $xy \neq 0$ 5+5

$= 0$, when $xy = 0$

7. a) Show that $\beta(m,n) = \int_0^\infty \dfrac{x^{m-1}}{(1+x)^{m+n}}dx$

Hence show that $\beta\left(\frac{1}{2},\frac{1}{2}\right) = \pi$.

b) Prove that, $\Gamma(n+1) = n\Gamma(n) = n!$

c) Show that the function defined by

$f(x) = \dfrac{1}{2^n}$, when $\dfrac{1}{2^{n-1}} < x \leq \dfrac{1}{2^n}$, for n = 0, 1, 2 ....

$= 0$, when $x = 0$,

is Riemann integrable on [0, 1]. 3+3+4

8. a) Test the convergence of the following :-

i) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x^2}}$, ii) $\displaystyle\int_0^\infty \frac{dx}{1+x^2}$

b) Prove that $\displaystyle\int_0^{\pi/2} \frac{\sin^m x}{x^n}dx$ exist if and only if $n < m+1$.

6+4

[Turn over