b) Find the modulus and the amplitude of the complex no.

$(2+3i)^2 /(3+4i)^3.$ 5+5

9. a) Simplify $(1+i)\left(1-\dfrac{1}{i}\right)$

b) Find the square root of $\pm i$. 5+5

10. a) Express $\dfrac{(Cos\theta + iSin\theta)6}{(Sin\theta + iCos\theta)5}$ in the form $A + iB$.

b) Find the values of $(1+i)^{\frac{1}{5}}$. 5+5

11. a) In a trapezium, prove that the straight line joining the mid-points of the diagonals is parallel to the parallel sides and half their difference.

b) Prove that for the vectors A, B, C

i) $[B \times C, C \times A, A \times B] = [ABC]^2$

ii) $A \times \{B \times (C \times D)\} = B \cdot D(A \times C) - B \cdot C(A \times D)$ 5+5

12. a) Find the magnitude and equation of the shortest distance between the lines $\dfrac{x}{2} = \dfrac{b}{-3} = \dfrac{z}{1}$ and $\dfrac{x-2}{3} = \dfrac{y-1}{-5} = \dfrac{z+2}{2}$

b) Find the distance of the point (1, –2, 3) from the plane $x- x - y + z = 5$ measured parallel to the line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{-6}.$ 5+5

BACHELOR OF ENGINEERING IN CHEMICAL ENGINEERING EXAMINATION, 2017

(1st Year, 2nd Semester, Old)

MATHEMATICS - IIB

Time : Three hours   Full Marks : 100

Answer *any ten (10)* questions :

1. a) Prove without expanding that

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$$

b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$

Given $x + y + z = 0$ 4+6

2. a) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

Prove.

[Turn over

b) Solve $\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0.$ 5+5

3. a) Find the adjugate and reciprocal determinant of

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

b) Solve by Cramer's rule

$$2x + y + 2z = 2$$
$$3x + 2y + z = 2$$
$$-x + y + 3z = 6$$ 5+5

4. a) Find A and B when

$$2A + B = \begin{bmatrix} 2 & 2 & 5 \\ 5 & 4 & 3 \\ 1 & 1 & 4 \end{bmatrix} \text{ and } A - 2B = \begin{bmatrix} 1 & 6 & 5 \\ 5 & 2 & -1 \\ -2 & -2 & 2 \end{bmatrix}$$

b) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -5 \\ -3 & 2 & 4 \end{bmatrix}$, find B, when $2A^T + 3B = 4I_3$

5+5

5. a) If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, them find A^2 and show that

$A^2 = A^{-1}.$

b) Solve by matrix method

$$x + y + z = 8$$
$$x - y + 2z = 6$$
$$3x + 5y - 7z = 14$$ 5+5

6. a) Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$

b) Find the value of a and b for which the system of equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b$$

have (i) no solution (ii) unique solution (iii) infinite no. (iv) solution. 5+5

7. Find the eigen values and the eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$ 10

8. a) Find the conjugate of complex no. $(2-i)/(1-2i)^2.$