BACHELOR OF ARCHITECTURE ENGINEERING EXAMINATION, 2017

(2nd Year, 1st Semester)

Mathematics - III A

Time : Three hours | Full Marks : 100
(50 marks for each part)

Use a separate Answer-script for each part.

PART - I

Answer any ***five*** questions.

All quesions carry equal marks.

1. (a) State and prove the law of addition of probability for any two events. 1+5

   (b) Two players A and B toss a die alternately. He who first throws a 'six' wins the game. If A begins the game, what is the probability of A's win? 4

2. (a) Define the following :

   (i) Mutually exclusive events, (ii) Mutually independent events, (iii) An exhaustive set of events. 2x3

   (b) If A and B are independent events and $P(A)=\frac{2}{3}$, $P(B)=\frac{3}{5}$, then find (i) $P(A\cup B)$ (ii) $P(A^c\cap B)$. 4

(Turn Over)

3. (a) Define probability mass function and probability density function. State their important properties. 6

(b) Show that f(x) defined by

$$f(x) = x,\ 0 \le x \le 1$$
$$= k - x,\ 1 \le x < 2$$
$$= 0,\ \text{elsewhere}$$

is a probability density function for a suitable value of the constant K. 4

4. (a) If a person gains or loses an amount equal to the number appearing when a perfect die is rolled once, according to whether the number is even or odd. How much money can he expect per game?

(b) If 20 dates are named at random, what is the probability that (i) 3 of them will be Sundays? (ii) 2 of them will be Sundays? 5+5

5. Describe Sun's motion along the ecliptic clearly stating Sun's co-ordinates at the equinoctial and solsticial points. 10

6. (a) State the four parts formula in spherical triangle. Deduce it from the cosine formula. 1+3

(b) If H be the hour angle of the sun at rising, show that

$$2\cos^2\left(\frac{H}{2}\right) = \sec\phi \sec\delta \cos(\phi + \delta)$$ 6

14. (a) In a simple random sample of n elements from a population of N elements, find the probability that a specific element is not included in the sample.

(b) Write down one advantage and one limitation of using simple random sampling.

(c) Find the curve for which cartesian subtangent is constant. 4+3+3

—— X ——

10. (a) Solve the following differential equation by the method of variation of parameters :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = xe^{-x}$$

(b) Solve : $\frac{dy}{dx} = \sin(x + 2y)$ 7+3

11. (a) Find the family of curves for which the angle between the radius vector and the tangent at (r, θ) is one half of the vectorial angle.

(b) Find the orthogonal trajectories of the family of hypo-cycloids $x^{2/3}+y^{2/3} = a^{2/3}$, where '$\eta$' is a variable parameter. 5+5

12. Show that the mean and standard error of sample mean ($\bar{X}$) for a sample of size 'n' are

$$E(\bar{X}) = \mu, \text{ S.E.of } \bar{X} = \sigma/\sqrt{\eta}$$

where μ, σ denote the mean and standard deviation of the population. 10

13. (a) In a stratified sampling, show that $\bar{X}_{st}$ is an unbiased estimate of the population mean 'μ' i.e. $E(\bar{X}_{st}) = \mu$, where $\bar{X}_{st}$ is the sample mean.

(b) What is meant by optimum allocation in stratified sampling? 7+3

7. (a) Define the following :
(i) Celestial meridian (ii) Ecliptic and (iii) The first point of Aries. 3x2

(b) If z be the zenith distance of a star of declination δ when on the prime vertical, prove that the latitude of the place is

$$\sin^{-1}(\sin\delta/\cos z)$$ 4

PART - II

Answer any ***five*** questions.

8. (a) Show that the differential equation

$$ydx + (x^2y - x)dy = 0$$

is not exact. Find an integrating factor and hence solve the differential equation.

(b) Solve the Bernoulli's equation

$$\frac{dy}{dx} - \frac{2y}{x} = x^2e^x$$ 5+5

9. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = e^{2x}$$

(b) If x^{α} be an integrating factor of

$$(x - y^2)dx + 2xydy = 0$$

then find α and hence solve it. 5+5