Bachelor of Engineering in Mechanical Engineering Examination, 2019

(4th Year, 1st Semester)

Finite Elements for Dynamics and Non-Linearity

Time : Three hours | Full Marks : 100

Answer ALL questions

1) Suppose you have a FE Package with you. Now answer **any three –** **(15)**

I. Bicycle –

What element do you choose to model it's frame?

During natural frequency determination, you are interested to account for effects of the tyre and a passenger. How do you propose to model them?

Suppose you are interested to find out the stress in the frame due to the passenger seated on it. How do you model the passenger now?

When the cycle is running on the road, to find the stresses due to the road undulation, what analysis do you suggest?

II. Transmission tower-

What element do you choose? What analysis do you need for finding its natural frequency?

The tower is subjected to self weight and pull due to the cables. What analysis do you suggest?

We are interested in safety of the tower during an earthquake. What analysis do you suggest to assure its safety.

III. Turbine blade (LP stage)-

What element do you choose? Suppose you want to find the natural frequency of the blade in non rotating condition. What analysis do you make?

Suppose you are interested in determining the effect of rotation on natural frequency. What additional effects do you need to account for? Will the frequency rise or fall?

[Turn over

IV. Rotor-

What element would you choose to model the rotor? In case the rotor has a crack which element would you suggest. How is a flexible bearing modeled? The critical speed of the rotor has to be found. What analysis do you suggest?

2) Answer the following question **(20)**

A beam of length 2L, density ρ, c/s area A and Young's modulus E is clamped at both . It is modeled by 2 equal beam elements. We need to find it's BENDING natural frequencies.

Write the [K] and [M] of either element using appendix. Assemble the Matrices. Solve for the bending natural frequencies.

3) Answer for **15 marks** –

(i) You need to find out the buckling load of a structure by FE analysis. Explain the steps you need to follow **(5)**

(ii) For a 1 dimensional element deduce the expression for geometrical stiffness matrix - $K_G = \int G^T PG\,dx$ where symbols have usual meaning **(5)**

(iii) Deduce any one element of the matrix K_G for a 2d beam element. **(10)**

Use Appendix

4) Answer for **50 marks** –

(i) Prove that – The eigen values of a real symmetric matrix are real. How does it help during determination of natural frequencies of non rotating structures. **(10)**

(ii) Explain the simultaneous iteration algorithm. **(10)**

(iii) Explain the Central difference scheme. What are its advantages and disadvantages **(10)**

(iv) Using Appendix, write the consistent mass matrix of a 2d bar element. Prove it. (Start from the shape function = $a_0 + a_1x$). Write the lumped mass matrix of the same element. **(10)**

(v) Write the expressions for Green Lagrange normal strain and engineering normal strain. For 1d condition, obtain the relation between them. **(5)**

(vi) You need to find the natural frequencies of a beam free on either end. Why do you need to shift. In case your program does not allow shifting, how would you model the beam **(5)**

(vii) Suppose you need to find the first two natural frequencies of system with 100 dof. Would you prefer Jacobi's method or Inverse iteration method. Explain. **(5)**

(viii) Diagonalise the following matrix without changing the eigen values

$$[A] = \begin{bmatrix} 4 & 3 \\ 3 & 8 \end{bmatrix} \qquad (5)$$

(ix) (xx) What is Rayleigh damping? Discuss its advantages in context to the Mode superposition method. Can it be implemented in the direct integration method. In a certain model, beside specification of material damping as Rayleigh damping, there is a concentrated damper at some of the nodes. Can both the above methods be used? **(10)**

APPENDIX – Information on 2d beam element

$$[K]=\begin{bmatrix} AE/l & 0 & 0 & -AE/l & 0 & 0 \\ 0 & 12EI/l^3 & 6EI/l^2 & 0 & -12EI/l^3 & 6EI/l^2 \\ 0 & 6EI/l^2 & 4EI/l & 0 & -6EI/l^2 & 2EI/l \\ -AE/ & 0 & 0 & AE/l & 0 & 0 \\ 0 & -12EI/l^3 & -6EI/l^2 & 0 & 12EI/l^3 & -6EI/l^2 \\ 0 & 6EI/l^2 & 2EI/l & 0 & -6EI/l^2 & 4EI/l \end{bmatrix}$$

$$[M]=\frac{\rho Al}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ 0 & 156 & 22l & 0 & 54 & -13l \\ 0 & 22l & 4l^2 & 0 & 13l & -3l^2 \\ 70 & 0 & 0 & 140 & 0 & 0 \\ 0 & 54 & 13l & 0 & 156 & -22l \\ 0 & -13l & -3l^2 & 0 & -22l & 4l^2 \end{bmatrix}$$

$$\lfloor N \rfloor = \left\lfloor 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad x - \frac{2x^2}{l} + \frac{x^3}{l^2} \quad \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad -\frac{x^2}{l} + \frac{x^3}{l^2} \right\rfloor$$