Ex./ARCH/MATH/T/124/2017

BACHELOR OF ARCHITECTURE EXAMINATION, 2017

(1st Year, 2nd Semester)

Mathematics - II

Time : Three hours Full Marks : 100

Use a separate Answer-Script for each part.

PART - I (30 marks)

Answer any ***three*** questions.

1. (a) Show that

$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & a^2 & b^2 \end{vmatrix} = \left(a^3 + b^3\right)^2$$

(b) Solve the equation

$$\begin{vmatrix} x+2 & 2x+3 & 3x+4 \\ 2x+3 & 3x+4 & 4x+5 \\ 3x+5 & 5x+8 & 10x+17 \end{vmatrix} = 0$$

5+5

2. (a) Solve by Cramer's rule

$x + 2y + 3z = 6$

$2x + 4y + z = 7$

$3x + 2y + 9z = 14$

(Turn over)

(b) Find the adjoint and the reciprocal determinant of

$$\begin{vmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix}.$$ 5+5

3. (a) Determine matrices A and B, where

$$A+2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \text{ and } 2A-B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

(b) If $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ the find the value of $(A^3 - A^2 - I)$.

6+4

4. (a) Compute the adjoint and the inverse of the matrix

$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{bmatrix}$ Also verify $AA^{-1} = I$.

(b) Find the rank of $\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}$. 6+4

9. Answer any ***two*** : 10x2=20

(a) Find the equation of the cone with vertex (α, β, γ) and guiding curve $ax^2 + by^2 = 1$, $z = 0$. Hence, show that locus of the points from which three mutually perpendicular lines can be drawn to intersect the conic $ax^2 + by^2 = 1$, $z = 0$ is $ax^2 + by^2 + (a+b)z^2 = 1$.

(b) Show that the equation $7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$ represents a cone whose vertex is (1,–2,2).

(c) If the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, then find equation of other two generators.

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(b) Find the equation of the plane bisecting the obtuse angle between the planes $4x - 3y + 12z + 13 = 0$ and $x + 2y + 2z = 9$.

(c) Find the magnitude and the position of the line of shortest distance between the lines $2x + y - z = 0 = x - y + 2z$ and $x + 2y - 3z - 4 = 0 = 2x - 3y + 4z - 5$.

(d) The sum of the squares of the intercepts on the co-ordinate axes made by a variable plane is equal to a constant k^2. Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right)\left(x^2 + y^2 + z^2\right)^2 = k^2$$

8. Answer any ***two*** : 8x2=16

(a) Find the equation of the circle passing through the points (2,0,1), (–2,1,0) and (0,3,5).

(b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle.

(c) Find the equation of the cylinder whose generators are parallel to the fixed line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and guiding curve is $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$.

5. Investigate the value of λ and μ so that the equations

$2x + 3y + 5z = 9$

$7x + 3y - 2z = 8$

$2x + 3y + \lambda z = \mu$

have (i) no solutions (ii) unique solution (iii) infinite number of solution.

Also solve when the system of equations have unique solution. 10

PART - II (70 marks)

Answer ***all*** questions.

6. (a) Find the radius of curvature, center of curvature and equation of the circle of curvature for the plane curve $x^2y + 2x + y = 6$ at the point (1,2).

(b) What do you mean by rectilinear asymptote of a plane curve ? (4+2+2)+2=10

7. Answer any ***three*** : 8x3=24

(a) Define the direction cosines of a line. If l, m, n are the direction cosines of a line, then using the definition prove $l^2 + m^2 + n^2 = 1$. 2+6