Ex./ARCH/MATH/T/124/2017

BACHELOR OF ARCHITECTURE EXAMINATION, 2017

(1st Year, 2nd Semester)

Mathematics - II

Time : Three hours

Full Marks : 100

Use a separate Answer-Script for each part.

PART - I (30 marks)

Answer any *three* questions.

1. (a) Show that

$$\begin{vmatrix} a^{2} & 2ab & b^{2} \\ b^{2} & a^{2} & 2ab \\ 2ab & a^{2} & b^{2} \end{vmatrix} = (a^{3} + b^{3})^{2}$$

(b) Solve the equation

2. (a) Solve by Cramer's rule

$$x + 2y + 3z = 6$$

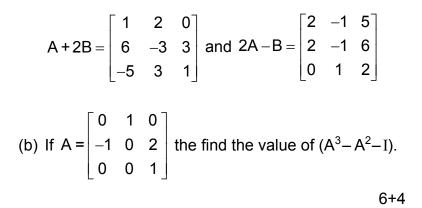
 $2x + 4y + z = 7$
 $3x + 2y + 9z = 14$

(Turn over)

- (b) Find the adjoint and the reciprocal determinant of
 - 0
 1
 2

 2
 0
 1

 1
 2
 0
- 3. (a) Determine matrices A and B, where



4. (a) Compute the adjoint and the inverse of the matrix

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 1 & -1 \end{bmatrix}$$
 Also verify $AA^{-1} = I.$
(b) Find the rank of
$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 2 & -2 & 2 \end{bmatrix}.$$
 6+4

9. Answer any *two* :

- (a) Find the equation of the cone with vertex (α , β , γ) and guiding curve $ax^2 + by^2 = 1$, z = 0. Hence, show that locus of the points from which three mutually perpendicular lines can be drawn to intersect the conic $ax^2 + by^2 = 1$, z = 0 is $ax^2 + by^2 + (a+b)z^2 = 1$.
- (b) Show that the equation $7x^2+2y^2+2z^2-10zx+10xy+26x-2y+2z-17=0$ represents a cone whose vertex is (1,-2,2).
- (c) If the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of a set of

three mutually perpendicular generators of the cone 5yz - 8zx - 3xy = 0, then find equation of other two generators.

____ X ____

- (b) Find the equation of the plane bisecting the obtuse angle between the planes 4x 3y + 12z + 13 = 0 and x + 2y + 2z = 9.
- (c) Find the magnitude and the position of the line of shortest distance between the lines 2x + y z = 0 = x y + 2z and x + 2y 3z 4 = 0 = 2x 3y + 4z 5.
- (d) The sum of the squares of the intercepts on the coordinate axes made by a variable plane is equal to a constant k². Show that the locus of the foot of the perpendicular from the origin to the plane is

$$\left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}\right) \left(x^2 + y^2 + z^2\right)^2 = k^2$$

8. Answer any *two* :

8x2=16

- (a) Find the equation of the circle passing through the points (2,0,1), (-2,1,0) and (0,3,5).
- (b) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y 2z + 2 = 0$, 2x + 3y + 4z = 8 is a great circle.
- (c) Find the equation of the cylinder whose generators are parallel to the fixed line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$ and guiding curve is $x^2 + y^2 + z^2 = 9$, x - y + z = 3.

5. Investigate the value of λ and μ so that the equations

$$2x + 3y + 5z = 9$$

 $7x + 3y - 2z = 8$
 $2x + 3y + \lambda z = \mu$

have (i) no solutions (ii) unique solution (iii) infinite number of solution.

Also solve when the system of equations have unique solution. 10

PART - II (70 marks)

Answer all questions.

- 6. (a) Find the radius of curvature, center of curvature and equation of the circle of curvature for the plane curve $x^2y + 2x + y = 6$ at the point (1,2).
 - (b) What do you mean by rectilinear asymptote of a plane curve ? (4+2+2)+2=10
- 7. Answer any *three* :

8x3=24

(a) Define the direction cosines of a line. If I, m, n are the direction cosines of a line, then using the definition prove $l^2 + m^2 + n^2 = 1$. 2+6

(Turn over)