Ex./ARCH/MATH/T/125/2017(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2017 (1st Year, 2nd Semester, Old Syllabus) Mathematics - II A

Time: Three hours Full Marks: 100

Answer any *ten* questions.

1. (a) If x+y+z=0, then show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$

(b) Prove without expanding

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$
5+5

2. (a) Solve

$$\begin{vmatrix} x^{3} - a^{3} & a^{2} & x \\ b^{3} - a^{3} & b^{2} & b \\ c^{3} - a^{3} & c^{2} & c \end{vmatrix} = 0$$

(Turn over)

(b) Show that

$$\begin{vmatrix} a^{2} & 2ab & b^{2} \\ b^{2} & a^{2} & 2ab \\ 2ab & b^{2} & c^{2} \end{vmatrix} = (a^{3} + b^{3})^{2}$$
5+5

3. (a) Find the adjugate and reciprocal determinant of

(b) Solve by Cramer's rule

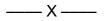
$$2x+y+2z = 2$$

 $3x+2y+z = 2$
 $-x+y+3z = 6$ 5+5

- 4. (a) If $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$ then show that $A^2 4A 5I = 0$,
 - (b) Verify that $[AB]^T = B^T A^T$, where

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 6 & -3 & 4 \end{vmatrix} \text{ and } B = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 5 & 6 & 1 \end{vmatrix}$$

- (b) If the normal to the hyperbola $xy = c^2$ at $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve the curve at $\left(ct_2, \frac{c}{t_2}\right)$ then $t_1^3 t_2 + 1 = 0$.
- 12. (a) Find the equation of the cylinder whose generating line is parallel to z-axis and the guiding curve is $x^2+y^2=z$, x+y+z=1.
 - (b) Find the equation of the right circular cylinder of radius 3 and whose axis is $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$. 5+5



(3)

- 8. (a) Find the equation plane parallel to the plane 2x-2y-z=3 and situated at a distance of 7 unit from it.
 - (b) Prove that the equation $2x^2 6y^2 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes. Find the angle between them. 5+5
- 9. (a) Find the equation of the plane passing through the three points (2,2,-1), (3,4,2) and (7,0,6).
 - (b) Find the equation of the plane which passes through the point (2,1,-1) and is orthogonal to the planes x-y+z=1 and 3x+4y-2z=0. 5+5
- 10. (a) Find the equation of the tangents to the conic $x^2+4xy+3y^2-5x-6y+3=0$, which are parallel to the straight line x+4y=0.
 - (b) Prove that if the straight line $\lambda x + \mu y + \gamma = 0$ touches the parabola $y^2 4px + 4pq = 0$, then $\lambda^2 q + \lambda y p\mu^2 = 0$. 5+5
- 11. (a) Show that the straight line lx+my=n is a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $\frac{a^2}{\ell^2} + \frac{b^2}{m^2} = (a^2 b^2)^2 / n^2$.

- 5. (a) If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$, then find A^{-1} .
 - (b) Find the rank of

6. Find for what value of a and b, the following system of equations

$$x+y+z=6$$

$$x+2y+3z=10$$

$$x+2y+az=b$$

have (i) no solutions (ii) unique solution (iii) an infinite no. of solutions. Also solve the system when it has unique solution.

7. Find the maximum value of z = 2x + 3y subject to the constraints :

$$x+y \le 30, \ y \ge 3,$$
 $0 \le y \le 12, \ x-y \ge 0$ and $0 \le x \le 20.$

(Turn over)

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