Ex./ARCH/MATH/T/125/2017(OLD)

BACHELOR OF ARCHITECTURE EXAMINATION, 2017

(1st Year, 2nd Semester, Old Syllabus)

Mathematics - II A

Time : Three hours | Full Marks : 100

Answer any ***ten*** questions.

1. (a) If $x+y+z=0$, then show that

$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$

(b) Prove without expanding

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

5+5

2. (a) Solve

$$\begin{vmatrix} x^3-a^3 & a^2 & x \\ b^3-a^3 & b^2 & b \\ c^3-a^3 & c^2 & c \end{vmatrix} = 0$$

(Turn over)

(b) Show that

$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & c^2 \end{vmatrix} = \left(a^3+b^3\right)^2$$ 5+5

3. (a) Find the adjugate and reciprocal determinant of

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

(b) Solve by Cramer's rule

$2x+y+2z = 2$

$3x+2y+z = 2$

$-x+y+3z = 6$ 5+5

4. (a) If $A = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix}$ then show that $A^2-4A-5I=0$,

(b) Verify that $[AB]^T = B^T A^T$, where

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 6 & -3 & 4 \end{vmatrix} \text{ and } B = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 2 \\ 5 & 6 & 1 \end{vmatrix}$$ 5+5

(b) If the normal to the hyperbola $xy=c^2$ at $\left(ct_1, \frac{c}{t_1}\right)$ meets the curve the curve at $\left(ct_2, \frac{c}{t_2}\right)$ then $t_1^3 t_2+1=0$. 5+5

12. (a) Find the equation of the cylinder whose generating line is parallel to z-axis and the guiding curve is $x^2+y^2=z$, $x+y+z=1$.

(b) Find the equation of the right circular cylinder of radius 3 and whose axis is $\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}$. 5+5

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8. (a) Find the equation plane parallel to the plane $2x-2y-z=3$ and situated at a distance of 7 unit from it.

(b) Prove that the equation $2x^2-6y^2-12z^2+18yz+2zx+xy=0$ represents a pair of planes. Find the angle between them. 5+5

9. (a) Find the equation of the plane passing through the three points (2,2,–1), (3,4,2) and (7,0,6).

(b) Find the equation of the plane which passes through the point (2,1,–1) and is orthogonal to the planes $x-y+z=1$ and $3x+4y-2z=0$. 5+5

10. (a) Find the equation of the tangents to the conic $x^2+4xy+3y^2-5x-6y+3=0$, which are parallel to the straight line $x+4y=0$.

(b) Prove that if the straight line $\lambda x+\mu y+\gamma=0$ touches the parabola $y^2-4px+4pq=0$, then $\lambda^2 q+\lambda y-p\mu^2=0$. 5+5

11. (a) Show that the straight line $lx+my=n$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, if $\frac{a^2}{\ell^2}+\frac{b^2}{m^2}=\left(a^2-b^2\right)^2/n^2$.

5. (a) If $A=\begin{vmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{vmatrix}$, then find A^{-1}.

(b) Find the rank of

$$\begin{vmatrix}1 & 2 & 3\\ 3 & 4 & 5\\ 2 & 1 & 1\end{vmatrix}$$

5+5

6. Find for what value of a and b, the following system of equations

$x+y+z=6$

$x+2y+3z=10$

$x+2y+az=b$

have (i) no solutions (ii) unique solution (iii) an infinite no. of solutions. Also solve the system when it has unique solution. 10

7. Find the maximum value of $z=2x+3y$

subject to the constraints :

$x+y\le 30,\ y\ge 3,$

$0\le y\le 12,\ x-y\ge 0$

and $0\le x\le 20$. 10